Whoever despises the high wisdom of mathematics nourishes himself on delusion.

Leonardo da Vinci

School of the Art Institute of Chicago

Calculus

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flash.uchicago.edu/~fxt/class_pages/class_calc.shtml

Syllabus

1	Aug 29	Pre-calculus
2	Sept 05	Rates and areas
3	Sept 12	Trapezoids and limits
4	Sept 19	Limits and continuity
5	Sept 26	Between zero and infinity
6	Oct 03	Derivatives of polynomials
7	Oct 10	Chain rule
8	Oct 17	Product rule and integrals
9	Oct 24	Quotent rule and inverses
10	Oct 31	Parametrics and implicits
11	Nov 7	Indefinite integrals
12	Nov 14	Riemann sums
13	Dec 05	Fundamental Theorem of Calculus

Sites of the Week

- www.calc101.com

- www.sosmath.com/calculus/diff/der00/der00.html

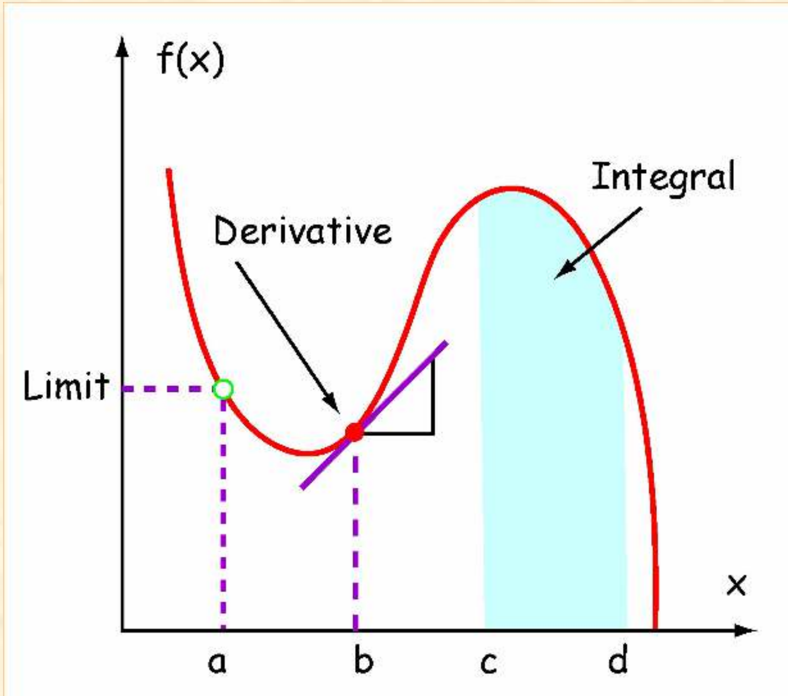
- www.sosmath.com/calculus/integ/integ01/integ01.html

Class #2

- Instantaneous rates
- The idea of integrals

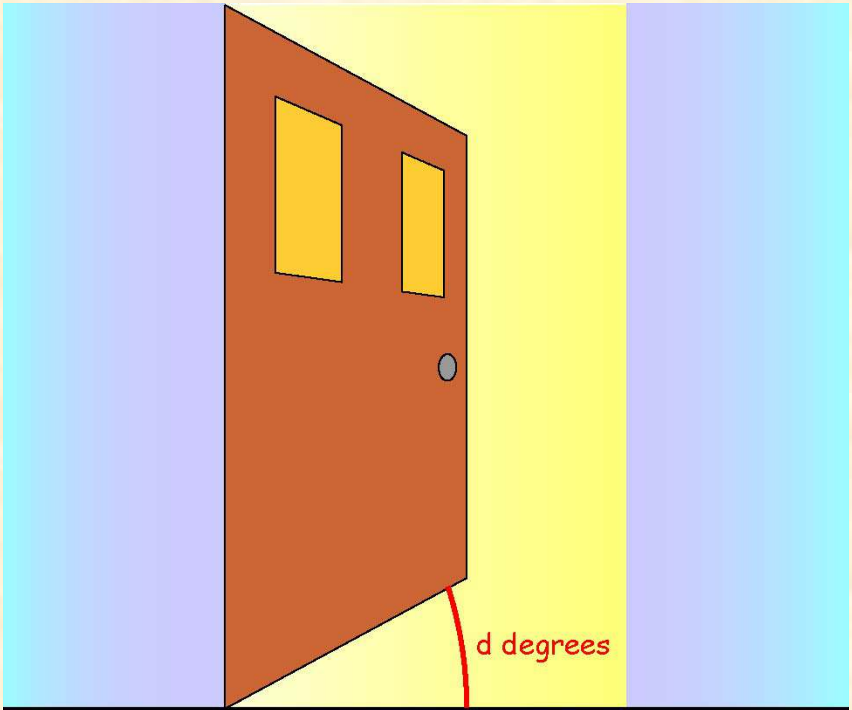
What is calculus?

- The basic ideas of calculus are limits, derivatives, and integrals.



Rates

- If we open a door with an automatic closer, it opens fast at first, slows down, stops, starts closing, then slams shut.



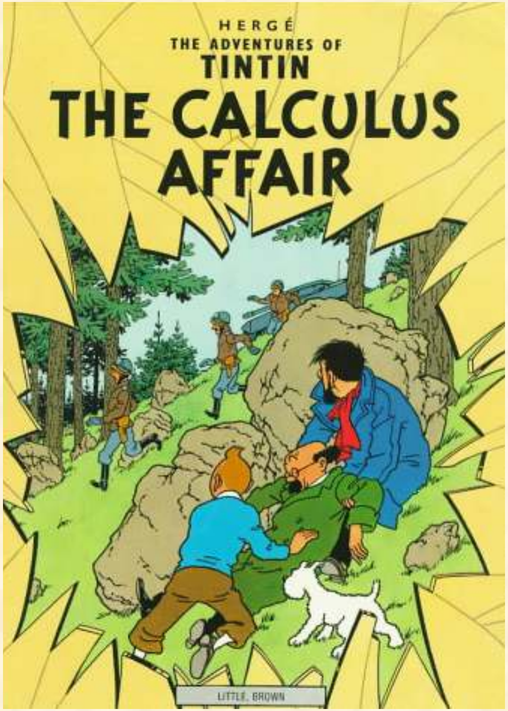
- As the door moves, the number of degrees it is from its closed position depends on how many seconds it has been since we first opened it.

Rates

- At any chosen time, is the door opening or closing and how fast is it moving?
- As we progress through this class, we'll learn to write exact equations expressing the rate of change of one variable quantity in terms of another.
- For the time being, we'll answer such questions graphically and numerically.

Objective

- Given the equation for a function relating two variables, estimate the instantaneous rate of change of the dependent variable with respect to the independent variable at a given point.

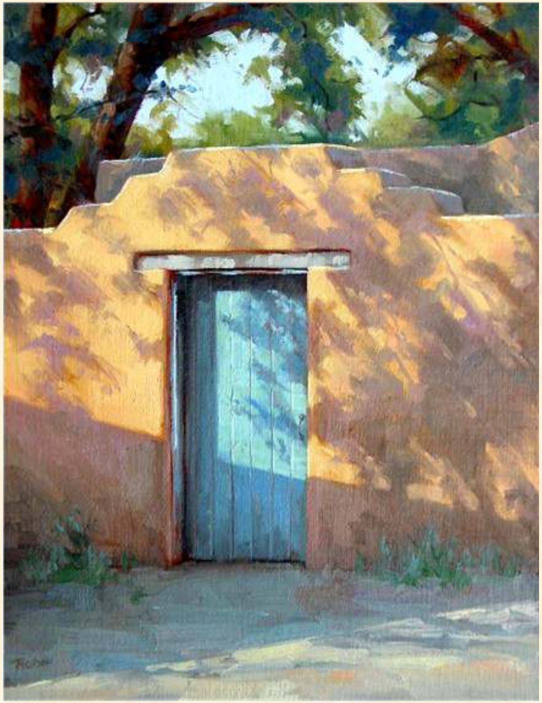


Rates

- Suppose the door is pushed open at time t = 0 sec and shuts again at t = 7 sec. When the door is in motion, assume that the number of degrees, d, from its closed position is modeled by:

$$d = 200\, t \cdot 2^{-t} \quad \text{for} \quad 0 \le t \le 7$$

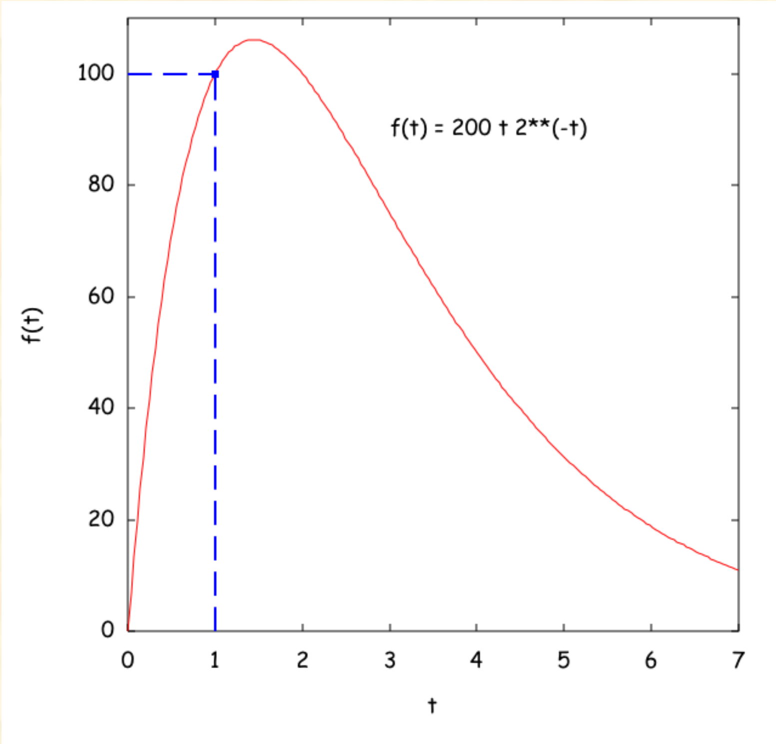
- How fast is the door moving at t = 1 sec?



Good Spirits May Enter
2001, Bob Rohm

Rates

- A graph of this equation is:



- When t = 1, the curve is increasing as t increases from left to right.
  So, the angle is increasing and the door is opening.

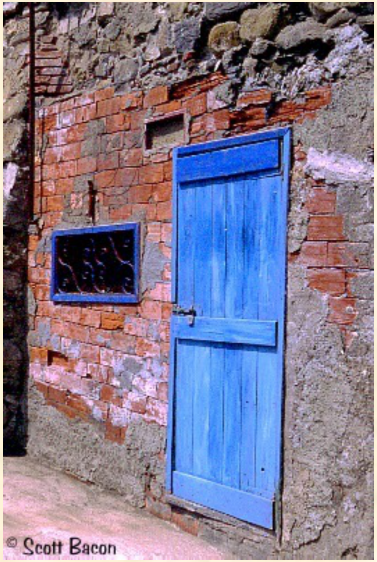
Rates

- We can estimate the rate the door is moving by calculating values of d for values of t close to 1.

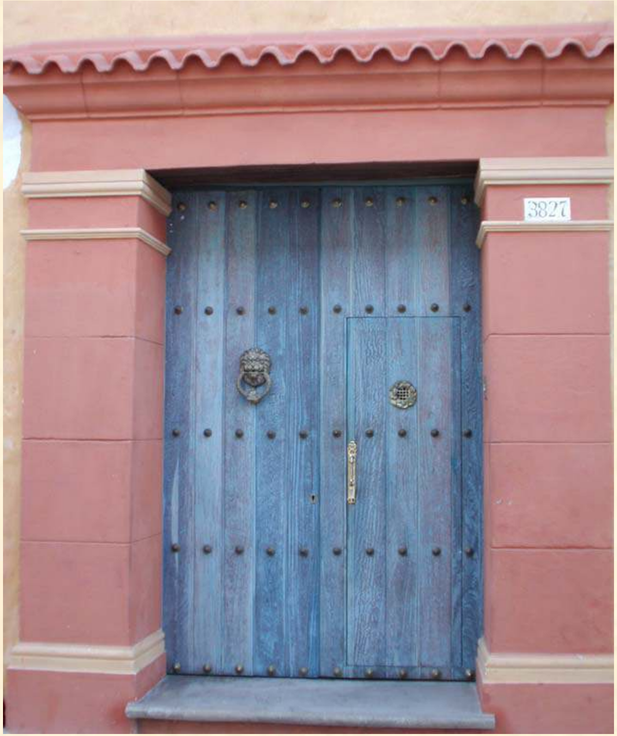
$t = 1$ $\quad\quad$ $d = 100°$

$t = 1.1$ $\quad\quad$ $d = 102.633629...°$

- The door's angle increased by about 2.633° in 0.1 sec, meaning that it moved at a rate of about 26.33 deg/sec.



Rates



- However, this 26.33 deg/sec is the average rate and the question asked about an instantaneous rate.

- In an "instant" that is 0 sec long, the door moves 0°.
  Thus the rate would be 0/0, which is undefined because of the division by zero.

Rates

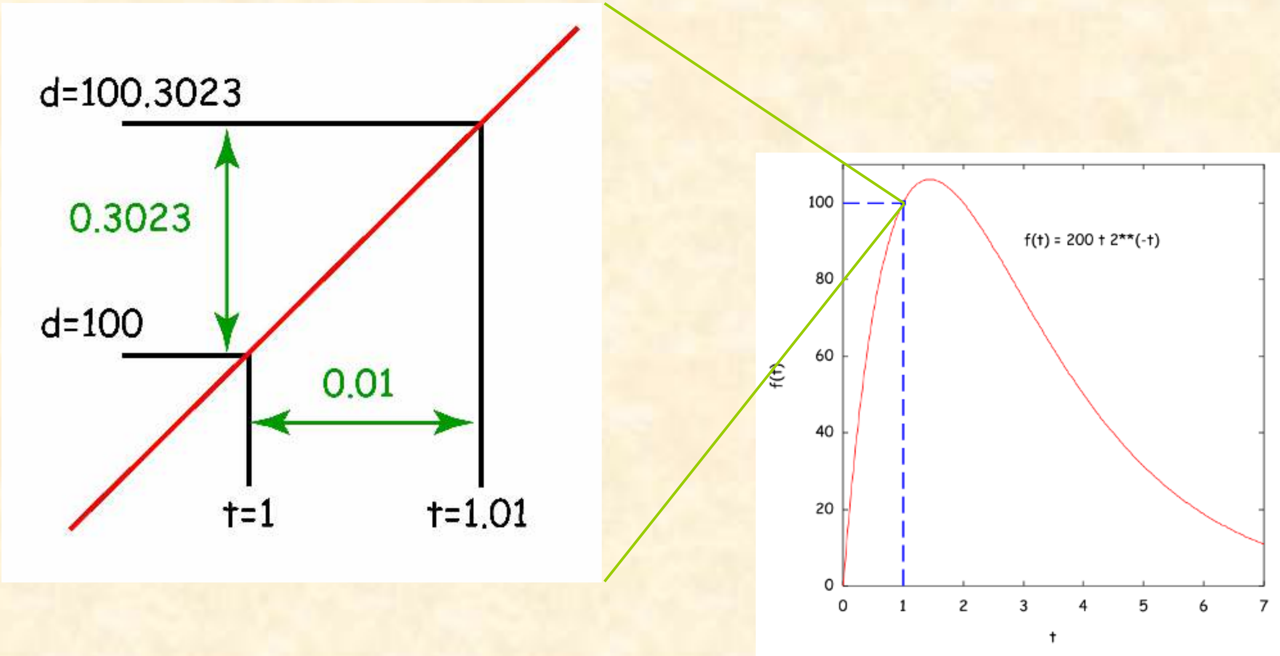
- To get closer to the instantaneous rate at t = 1 sec,
  find d at t = 1.01 sec and at t = 1.001 sec.

Time	Degrees	Rate at t = 1
1	100°	
1.1	102.633620…°	2.633/0.1 = 26.33 deg/sec
1.01	100.30234°	0.30234/0.01 = 30.234 deg/sec
1.001	100.03064…°	0.03064/0.001 = 30.64 … deg/sec

- Notice as the time interval gets smaller and smaller,
  the number of degrees per second doesn't change as much.

Rates

- As we zoom in on the point (1,100) the curve appears to get straighter, so the change in d divided by the change in t becomes closer to the slope of a straight line.



Rates

- If we list more average rates in a table,
  the values stay the same for more and more decimal places.

Time interval	Average rate
1 to 1.01	30.23420
1 to 1.001	30.64000
1 to 1.0001	30.68075
1 to 1.00001	30.68482
1 to 1.000001	30.68524

- There seems to be a limiting number that the values are approaching.

Rates

- Estimating the instantaneous rate at t = 3 sec gives the following results.

Time interval	Average rate
3 to 3.1	-26.899
3 to 3.01	-26.978
3 to 3.001	-26.985

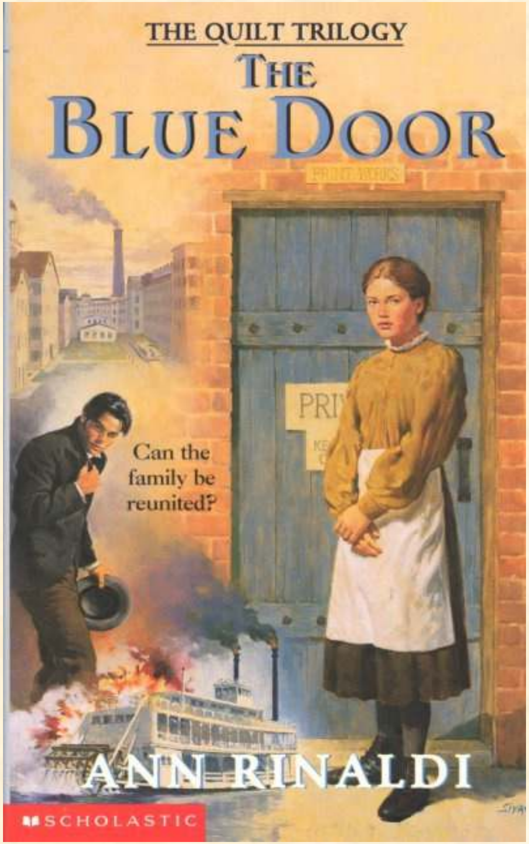
- Again the average rates seem to be approaching some limiting number, so the instantaneous rate should be somewhere close to -27 deg/sec.

- The negative sign tells us the number of degrees is decreasing as time goes on. Thus, the door is closing at t=3 sec.

Rates

- For our door example, the angle is said to be a function of time.
  Time is the independent variable and the angle is the dependent variable.

- These names make sense, because the number of degrees the door is opened depends on the number of seconds since it was pushed.



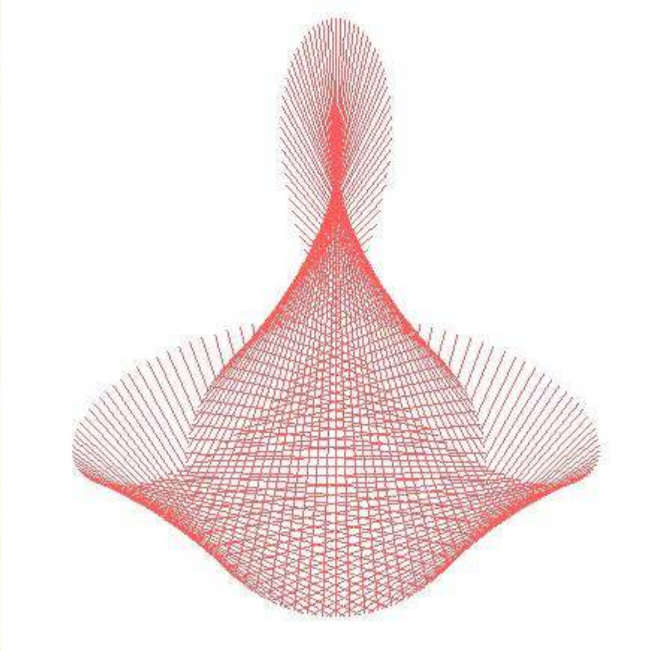
Rates

- The instantaneous rate of change of the dependent variable is said to be the limit of the average rates as the time interval gets closer to zero.

- This limiting value is called the derivative of the dependent variable with respect to the independent variable.

Meaning of a derivative

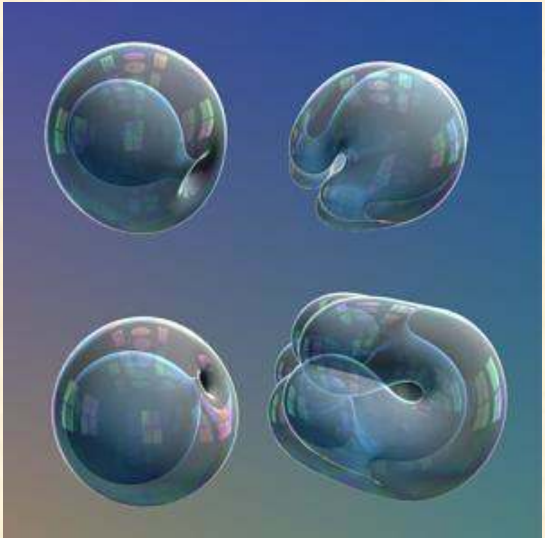
- The derivative of a function is the instantaneous rate of change of the dependent variable with respect to the independent variable.



Limit preview

- Verbal definition of a limit:

  L is the limit of f(x) as x approaches c
  if and only if
  L is the one number you can keep f(x) arbitrarily close to,
  just by keeping x close enough to c, but not equal to c.

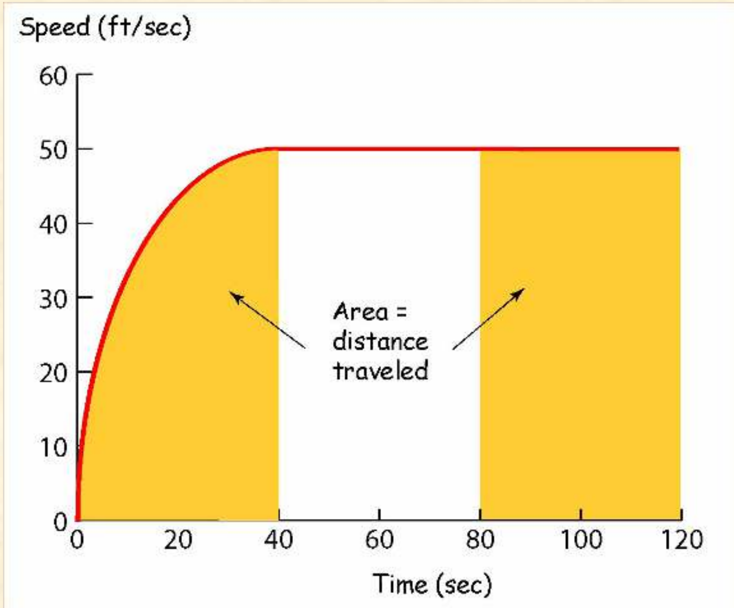


Interlude

The Class of 1892's Calculus Play, June 21, 1890. The play was an outgrowth of the Lafayette tradition of the cremation of calculus, in which students symbolically (and sometimes literally) burned their hated calculus texts.

Integrals

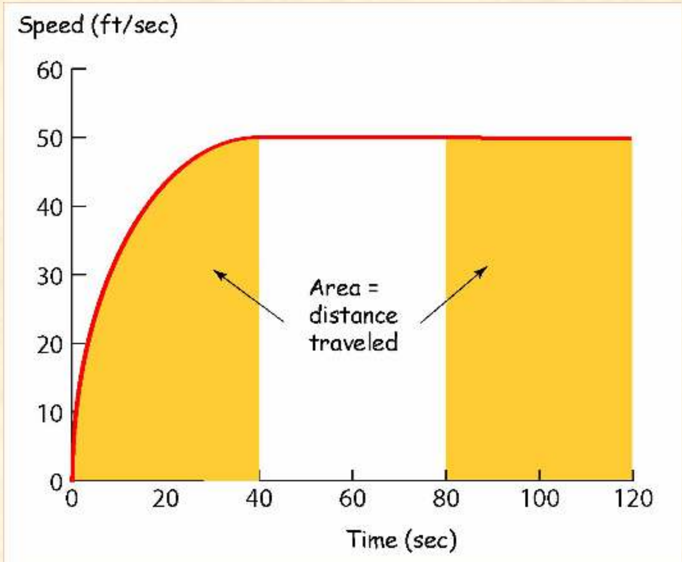
- Take off in your car. Your speed increases for awhile, then levels off.



- Between t = 40 and t = 80, the speed is a constant 50 ft/sec.
  Since distance = speed • time, you travel is 50 ft/sec • 40sec = 2000 ft.

Integrals

- Geometrically, 2000 ft is the area of the white rectangle. The width is 40 sec and the length is 50 ft/sec.

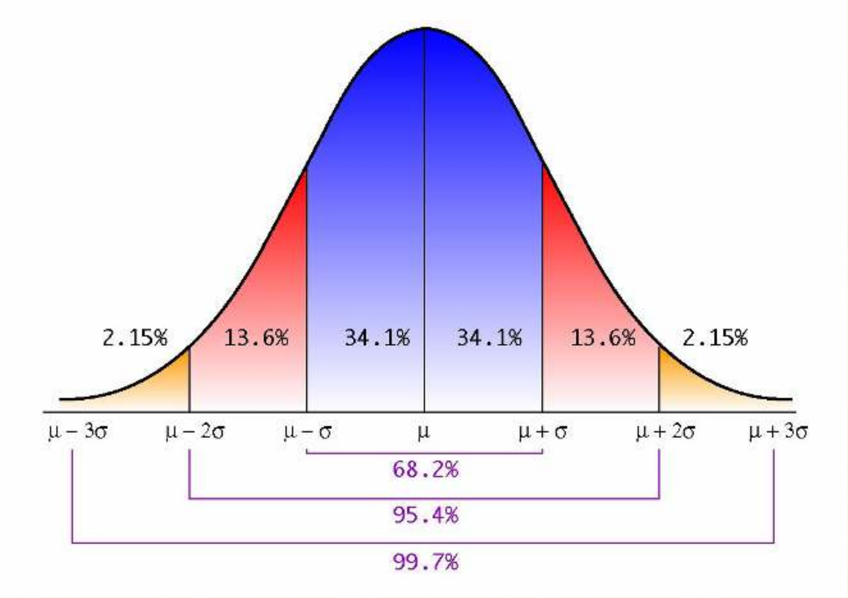


- Between 0 and 30 sec, when the velocity is changing, the area under the curve also equals the distance traveled. But because the length varies, the area cannot be found by multiplying two numbers.

Integrals

- The process of evaluating a product in which one factor varies is called finding a definite integral.

- Definite integrals can be evaluat
by finding the corresponding are



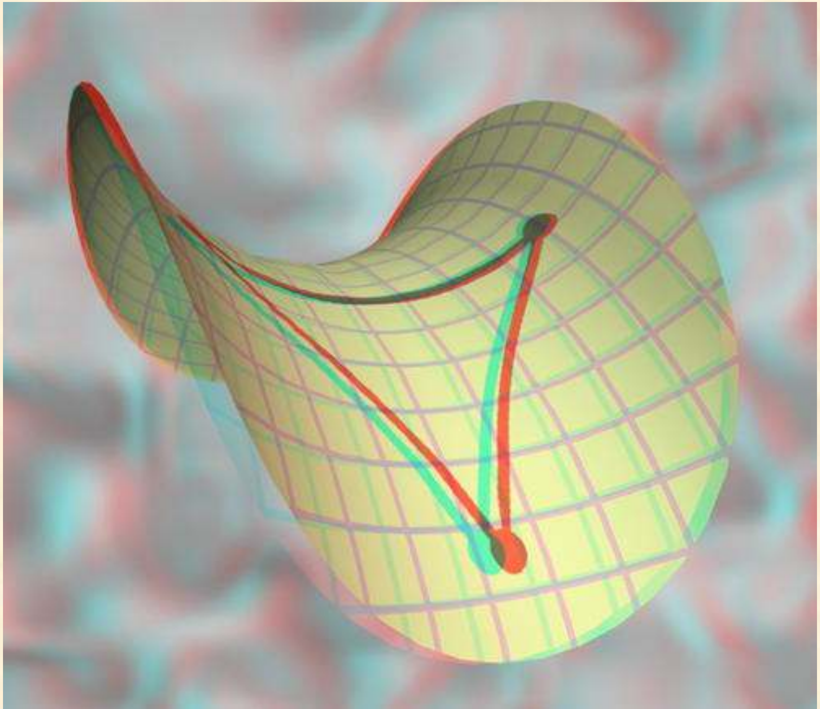
Integrals

- For now, we'll find approximate areas by counting squares ("brute force").

- Later in this class, we'll apply the concepts of limits to calculate exact values of definite integrals.

Objective

- Given the equation of graph for a function, estimate the definite integral of the function between x = a and x = b by counting squares.



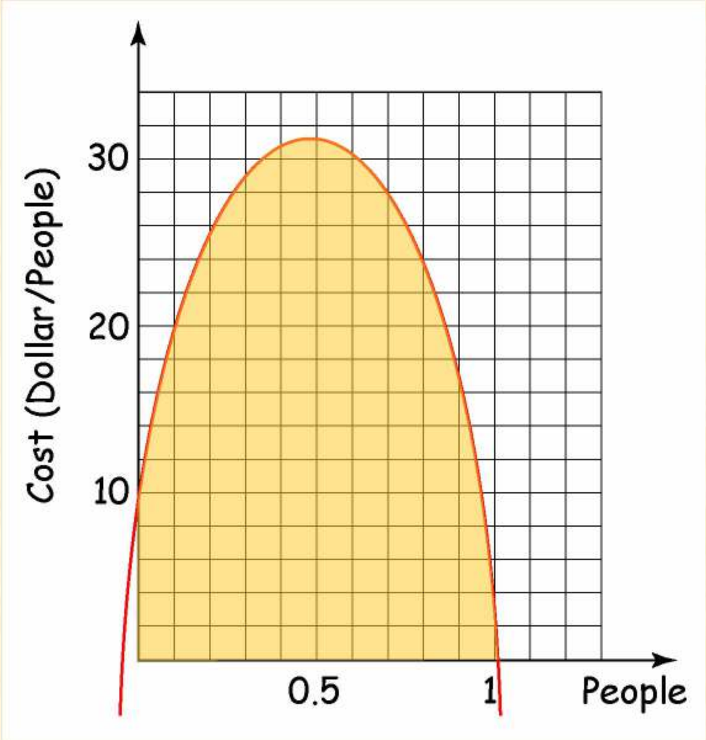
Integrals

- Estimate the definite integral of the cost function $c(p) = -100p^2 + 90p + 11$ from x=0 to x=1, where p is the number of people and c is the price per person.

- Each space in the people direction is 0.1 and each space in the cost direction is 2 dollars/person.

- So, each square represents $0.1 \cdot 2 = 0.2$ dollars.

- I count about 113.5 squares, so the definite integral is about $113.5 \cdot 0.2 = 22.7$ dollars.



Playtime

- During our in-class problem solving session today we'll estimate derivatives by average rates of change and definite integrals by counting squares.