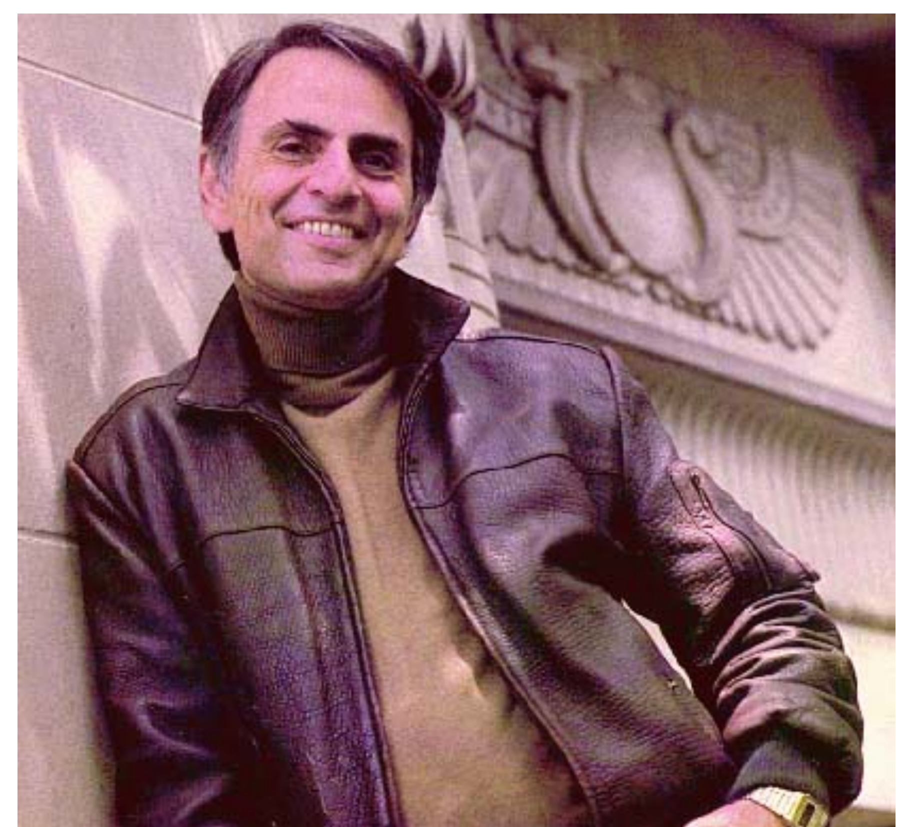
Coal, oil and gas are called fossil fuels, because they are mostly made of the fossil remains of beings from long ago. The chemical energy within them is a kind of stored sunlight originally accumulated by ancient plants. Our civilization runs by burning the remains of humble creatures who inhabited the Earth hundreds of millions of years before the first humans came on the scene. Like some ghastly cannibal cult, we subsist on the dead bodies of our ancestors and distant relatives.

Carl Sagan



Arizona State University
SES 194

Energy in Everyday Life

Order of Magnitude Estimate

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What is the energy in a hurricane?

Our guidelines for making an order-of-magnitude estimate:

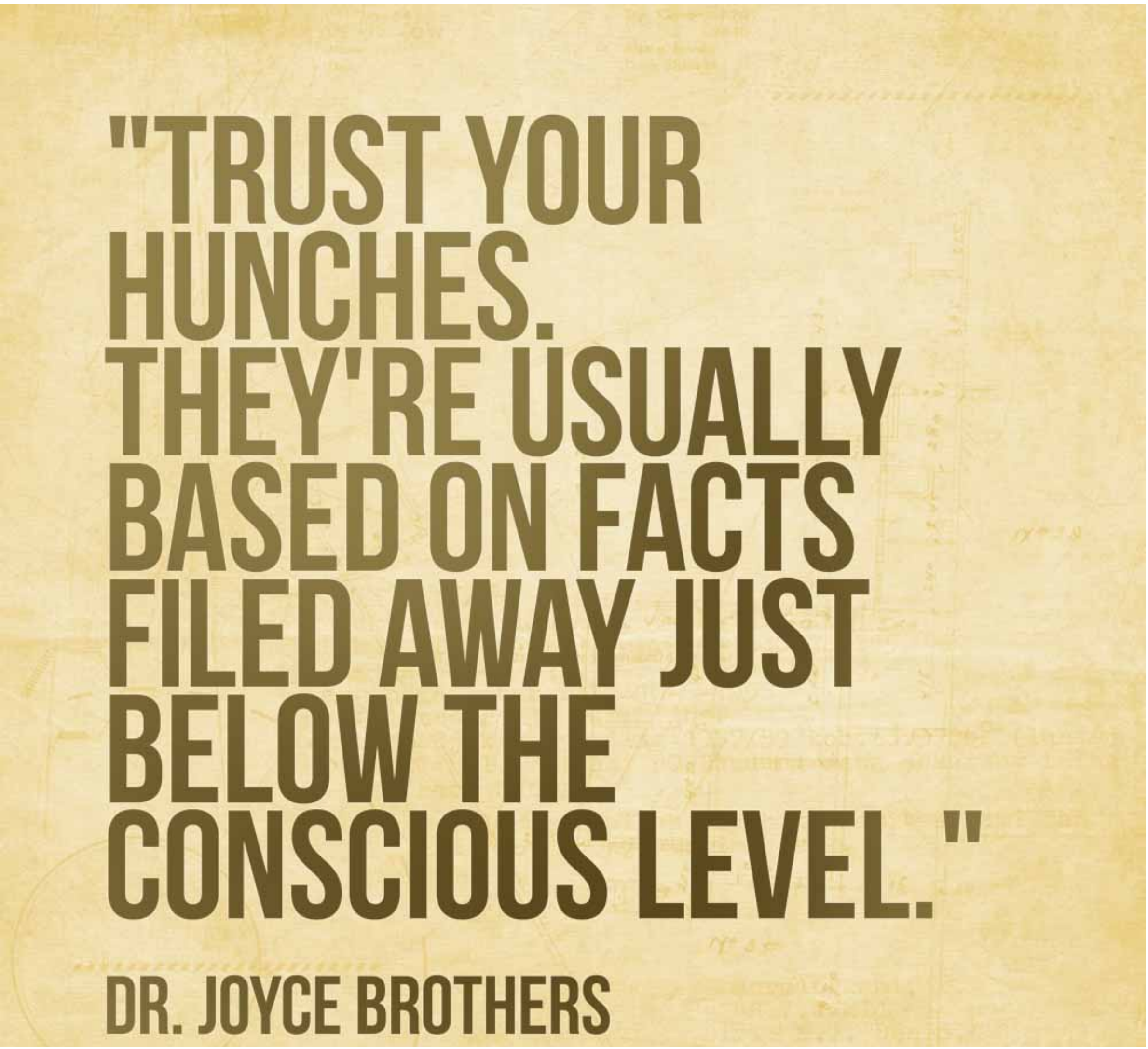
- ***Guess***
- ***Talk to your gut***
- ***Divide and conquer***
- ***Lie skillfully***
- ***Punt***
- ***Use guerrilla warfare***
- ***Lower your standards***
- ***Cross-check***

Guess

I know the annual US energy consumption is 10^{20} J, so i'll guess one hurricane has about the same energy.

A hurricane is ultimately driven by capturing solar energy.

I know at the surface of the Earth the power delivered by the Sun is ~ 1000 W/m^2.



A hurricane is a heat engine.

Evaporation of warm water from the ocean at ~25 °C (~300 K), condenses into clouds in the troposphere at ~-70 °C (~200 K).

Common Knowledge

Share what you know.

The efficiency of this heat engine is
$\varepsilon = (T_{hot} - T_{cold})/T_{hot} \sim (300 - 200)/300 \sim 1/3$.

Of the ~ 1000 W/m^2 delivered by the Sun,
~1/3 is available to the hurricane, ~ 300 W/m^2.

A hurricane can cover a good sized state, so i’ll assume a radius of ~ 200 miles, ~ 300 km.

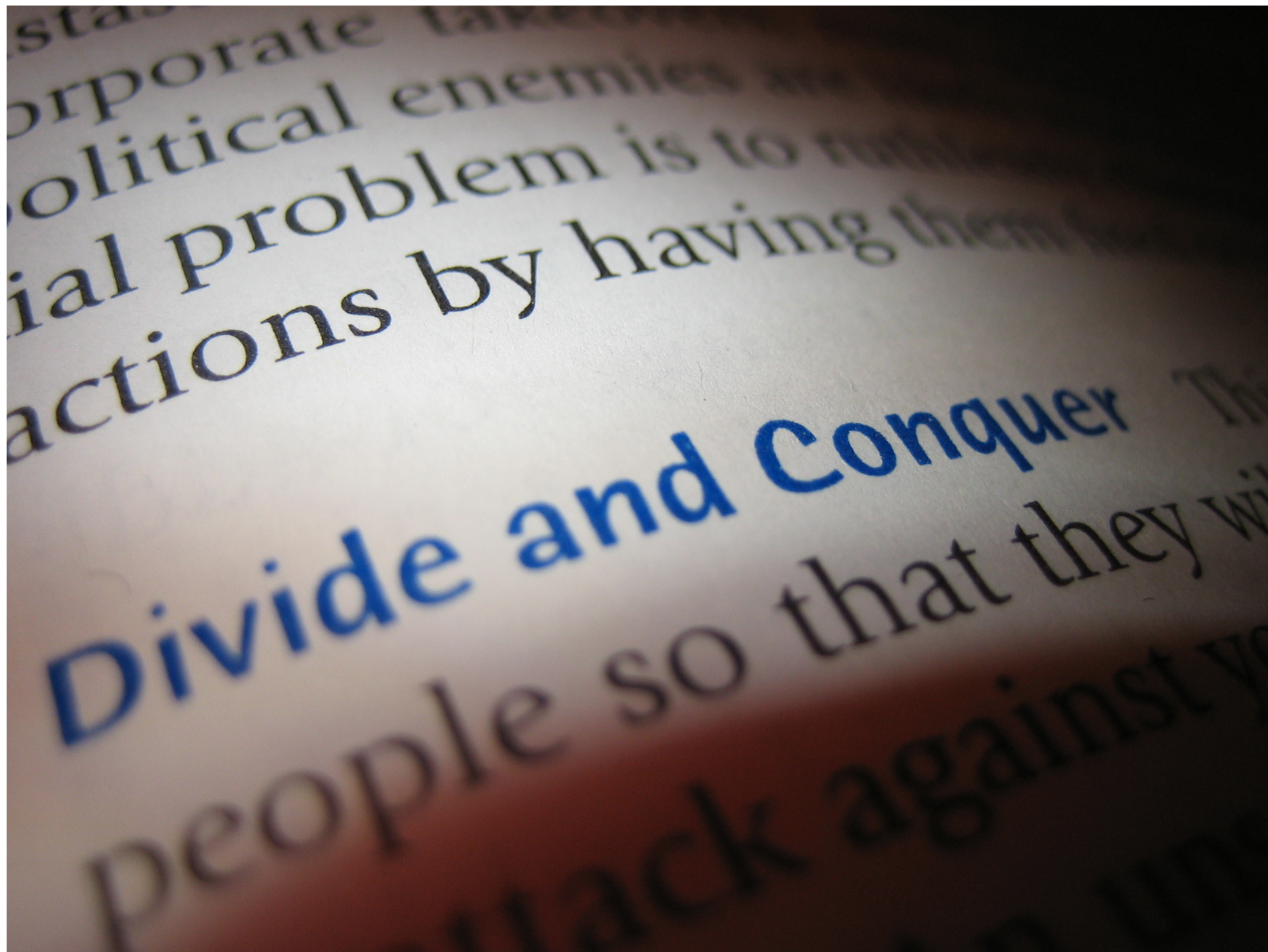
This corresponds to an area of
$\pi \times (3 \times 10^5 m^2) \sim 3 \times 3 \times 10^{10} \sim 10^{11} m^2$.

From news reports, I know a full strength hurricane lasts longer than 1 day but less than 10 days. I’ll assume 3 days, or $3 \times 24 \times 60 \times 60 \sim 10^5$ s.

A hurricane thus has a total energy of
~ (300 W/m^2) x (10^{11} m^2) x (10^5 s) ~ 3 x 10^{18} J

A hurricane's energy consists of the energy of motion of its winds (1/2 x mass x speed^2) and the gravitational potential energy of the water stored in its clouds (mass x gravitational acceleration x height) which is released when the rain falls.

I need to estimate a speed, air mass, rain mass and height of a hurricane.



A hurricane's peak wind speeds are ~ 100 mph ~ 50 m/s.
Punt on the (larger) parts that have tropical winds.

To get the hurricane's air mass we can multiply the mass density of air (~ 1 kg/m^3) times the volume of the hurricane.

Hurricane strength winds cover a large city,
so ~ 50 mile radius ~ 75 km radius. The area covered is
π x $radius^2$
~ 3 x $(75 \times 10^3)^2$
~ 3 x 75^2 x 10^6
~ 3 x 5000 x 10^6
~ 2 x 10^9 m^2.

A hurricane's height is up in the troposphere, so more then 1 km and less than 10 km, so i'll guess 3 km.

The volume of the hurricane winds is
Area x height ~ $(2 \times 10^9\ m^2) \times (3000\ m) \sim 10^{14}\ m^3$.

So the mass of the hurricane's air is
mass density x volume
~ $(1\ kg/m^3) \times (10^{14}\ m^3)$
~ 10^{14} kg.

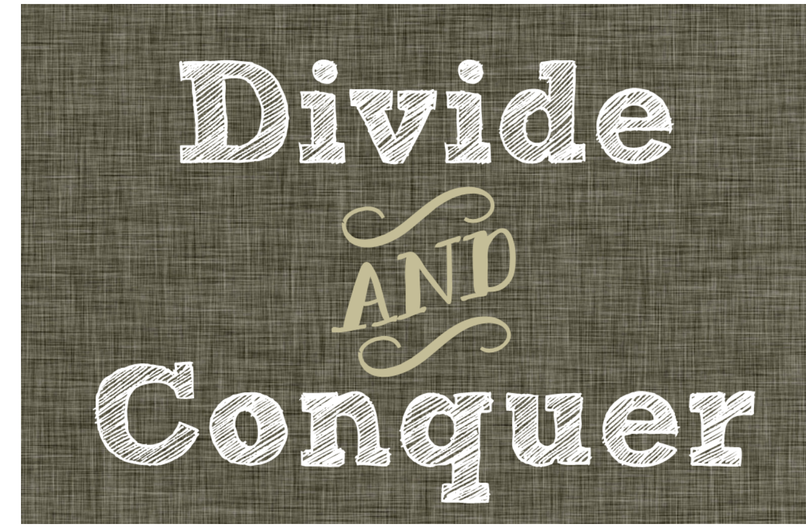
Hurricane's energy of motion is then

1/2 x mass x speed^2

~ 1/2 x (10^{14} kg) x (50 m/s)^2

~ 1/2 x 10^{14} x 2500

~ 10^{17} J



For the hurricane's rain mass, we can multiply the mass density of water (~ 1000 kg/m^3) times the volume covered by the rain.

I'll assume the rain falls in twice the radius, four times the area, as the winds, ~ 10^{10} m^2.

I recall the rains leave ~ 1 foot, ~1/3 meter, of water.

Common Knowledge
Share what you know.

So the volume of rain is
$\sim 10^{10}\ m^2 \times 1/3\ m$
$\sim 3 \times 10^9\ m^3$.

Thus the mass of the hurricane's rain is
mass density x volume
$\sim (1000\ kg/m^3) \times (3 \times 10^9\ m^3)$
$\sim 3 \times 10^{12}\ kg$.

Hurricane's gravitational potential energy release is then mass x gravitational acceleration x height
~ (3 x 10^{12} kg) x (10 m/s^2) x (3000 m)
~ 10^{17} J.

The energy released by the falling rain is at least as large as the wind energy.

Summing, the energy of motion and potential energy release, a hurricane's energy ~ 3 x 10^{17} J.

This is within a factor of 10 of my simpler estimate. Combing the two, the energy of a hurricane is ~ 10^{18} J.

Energy: 110 Orders of Magnitude