Where there is matter, there is geometry.

Johannes Kepler

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Geometry of Art and Nature

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flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

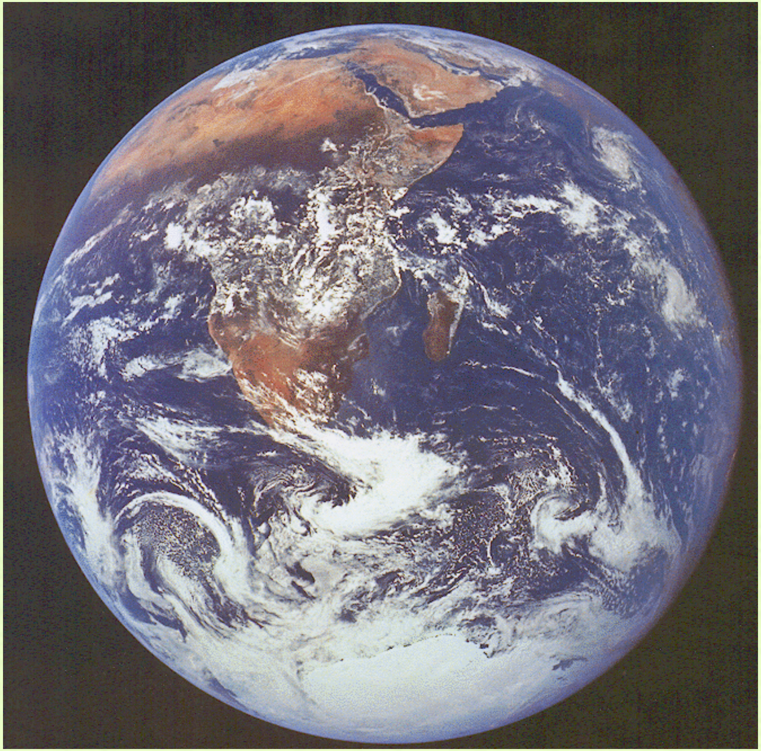
- graphicsgoddess.home.att.net/celtknottut.htm
- birrell.org/andrew/knotwork/

Class #1

- Those Pythagoreans
- Perimeter, Area, and Angle
- Polygons
- Celtic Knots

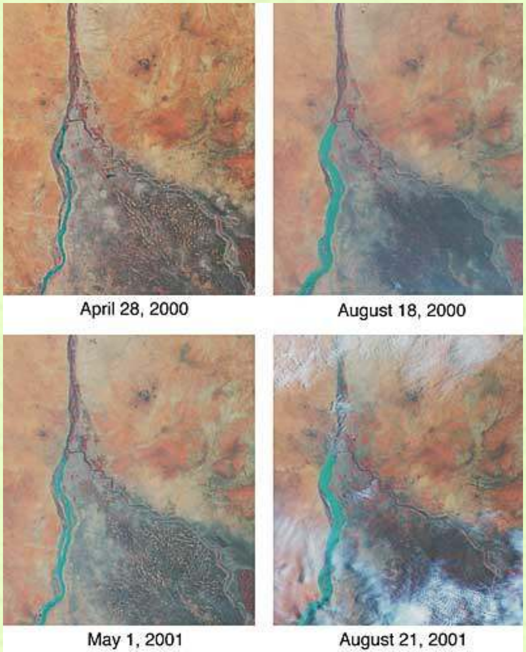
Earth measurement

- The word geometry comes from the Greek words for earth and measurement.



Earth measurement

- The first known application of *geometry* was surveying in Egypt, where annual flooding of the Nile eradicated the demarcations between fields.



Throughout history, the rising and falling waters of the mighty Nile River have directly affected the lives of the people who live along its banks.

Thousands of people in the Sudan lost their homes in 2001 to the swollen waters of the Nile, which reached their highest levels in more than two decades.

Earth measurement

- Geometry provided the survival tools needed to accurately and efficiently redivide the land into individual holdings after the floods.

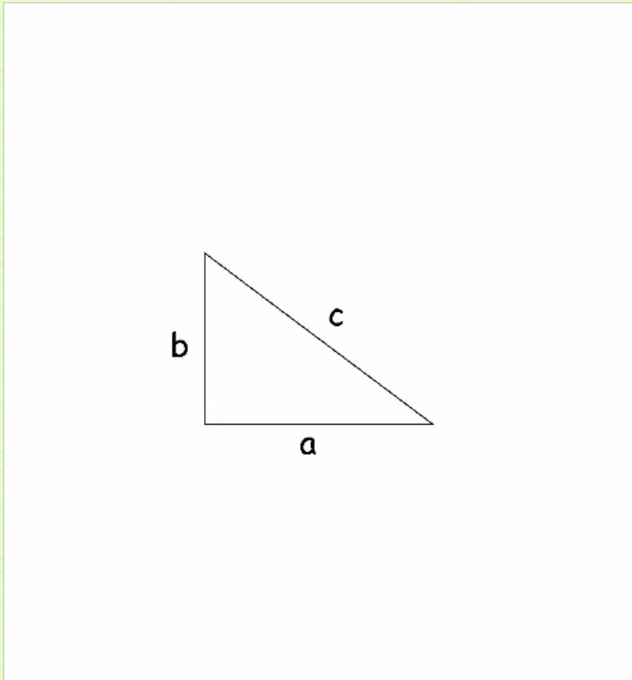
Earth measurement

- The Greeks developed the theoretical underpinnings of geometry, cumulating in Euclid's The Elements, circa 300 BC.

School of Athens,
1510, Raphael Sanzio

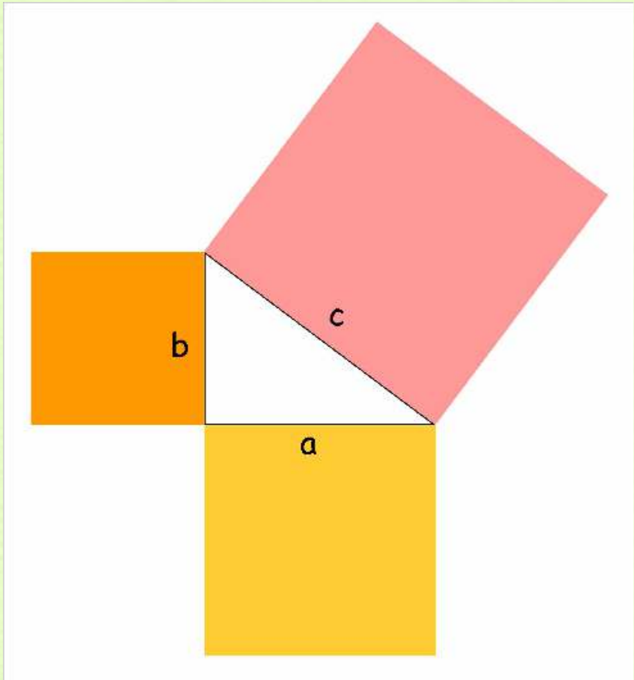
Pythagorean Theorem

- One of the most useful tools in geometry is the Pythagorean Theorem: If a right triangle has sides of lengths a, b, and c, then the lengths are related by $c^2 = a^2 + b^2$.



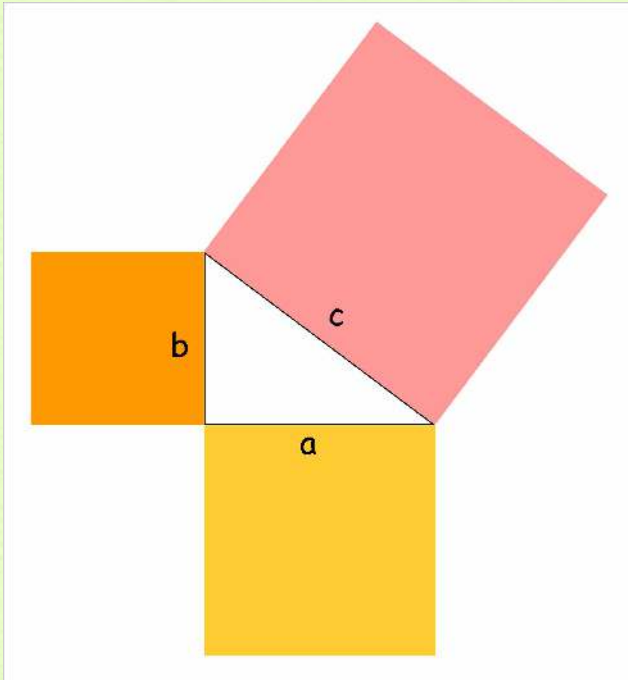
Pythagorean Theorem

- A way of stating the Pythagorean theorem in purely geometric terms is that the area of the square built with side c is equal to the combined areas of the squares built with sides a and b.



Pythagorean Theorem

- The converse of the Pythagorean theorem is also true:
  If a triangle has sides a, b, and c, and $c^2 = a^2 + b^2$, then the triangle is a right triangle with the right angle opposite the side c.

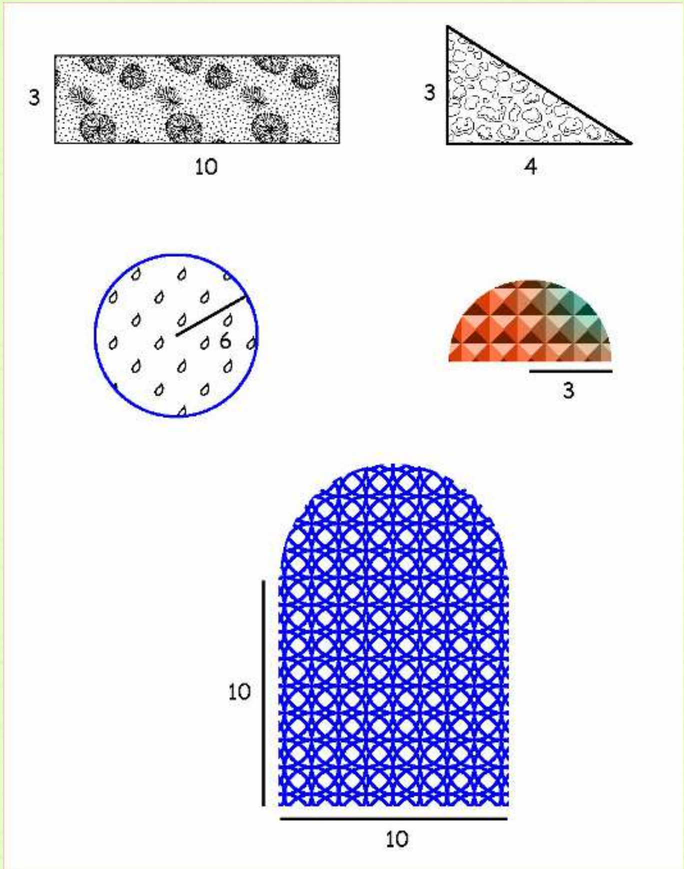


- If a triangle has sides of length 3, 4, and 5, is it a right triangle?
- If so, where is the right angle?

Perimeter

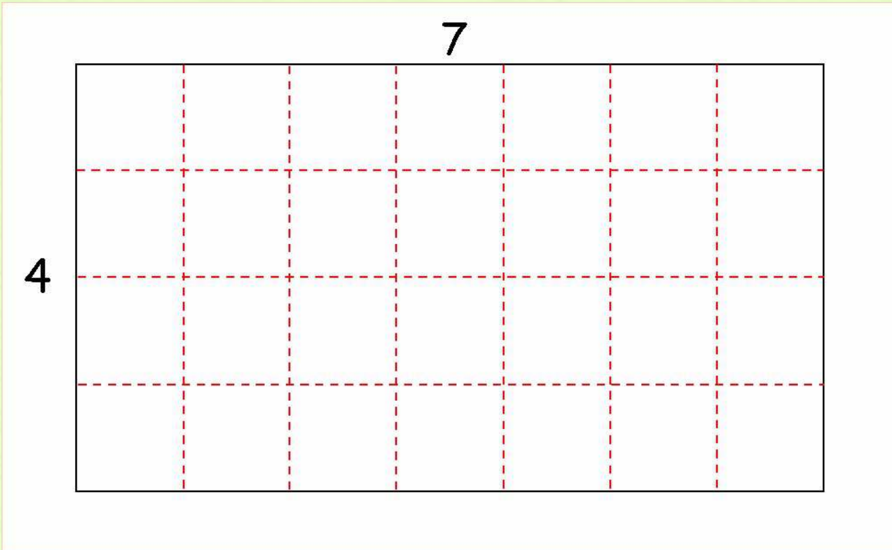
- The perimeter, or circumference, of a figure is the distance to walk all the way around the figure.

- For circles, one can use $P = 2 \pi r$ to figure out the perimeter, where r is the radius of the circle.



Area

- The figure whose area is easiest to measure is the rectangle.

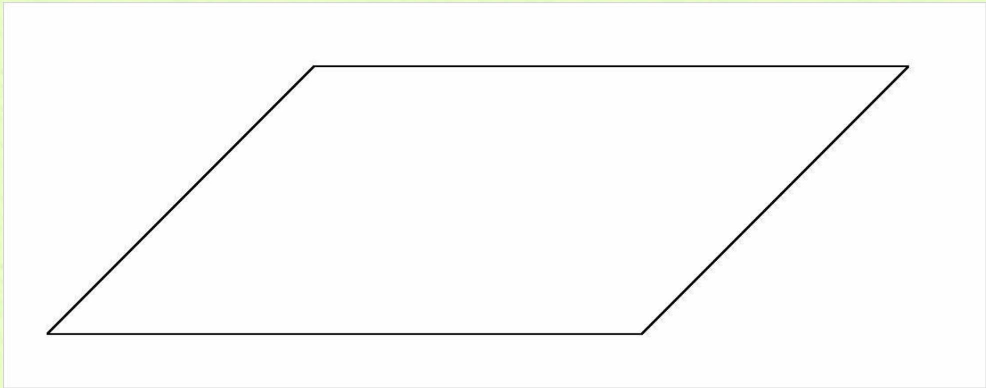


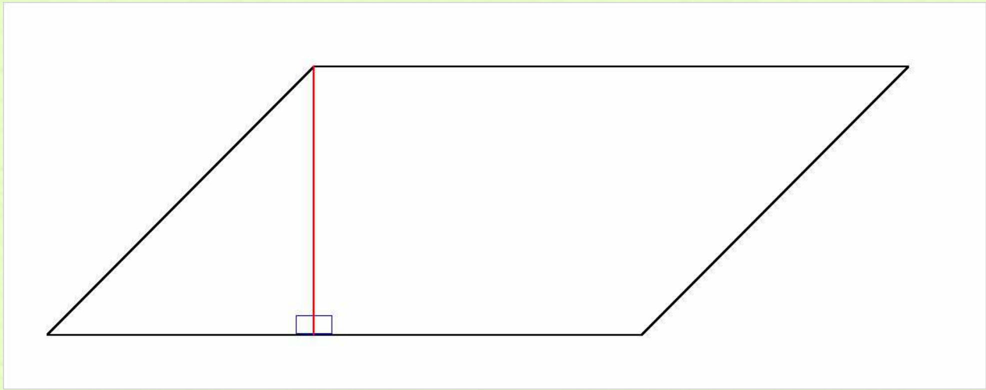
- Thus, the area of a rectangle is always the width times the height:

$$A \text{ (rectangle)} = w \cdot h$$

Area

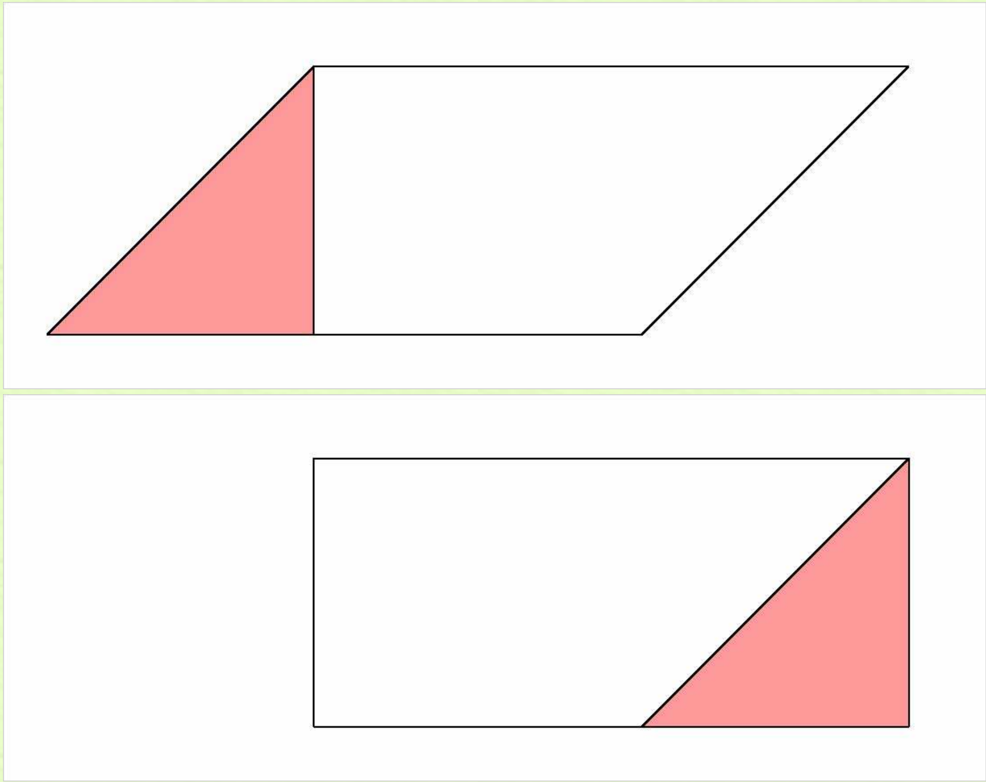
- We can figure out the area of a parallelogram by dropping a perpendicular line from the corner down to the other side.





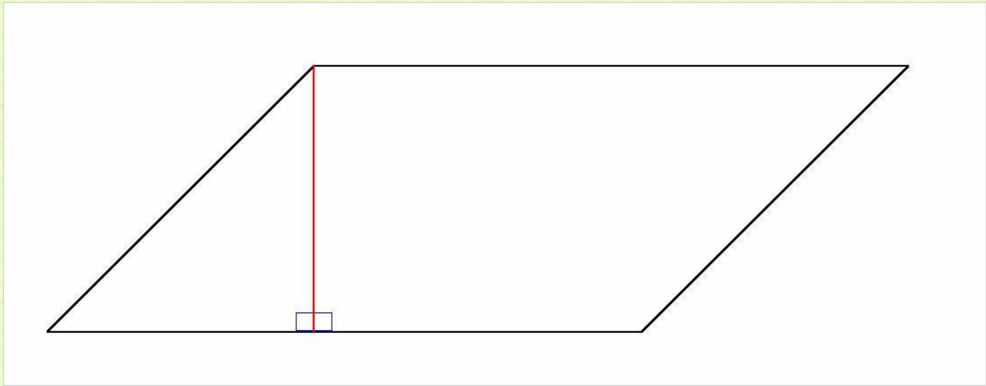
Area

- If we cut off the triangle portion and then put it the opposite side, a rectangle is formed that has the same area as the original parallelogram.



Area

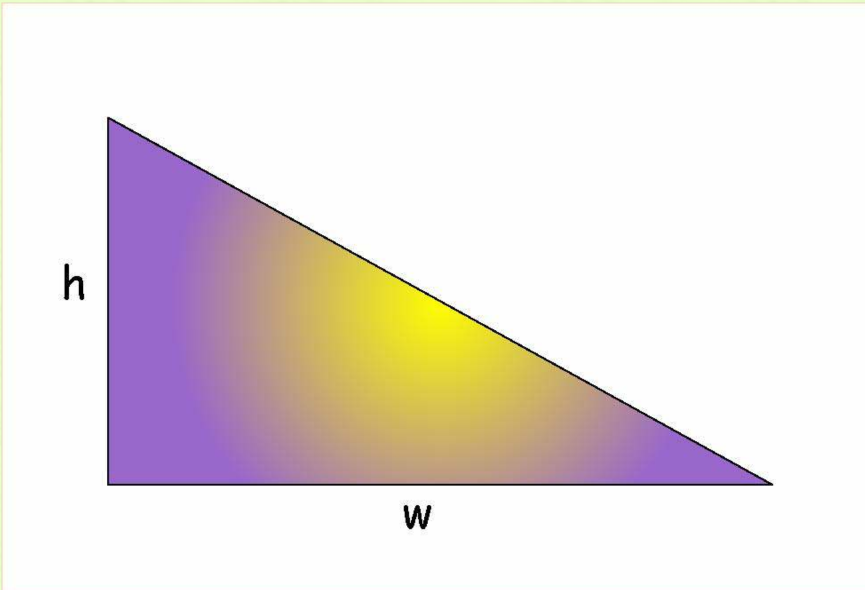
- Thus, the area of a parallelogram is also its width times height, with the height measured by dropping a perpendicular to the side that measures the width.



- A (parallelogram) = w · h

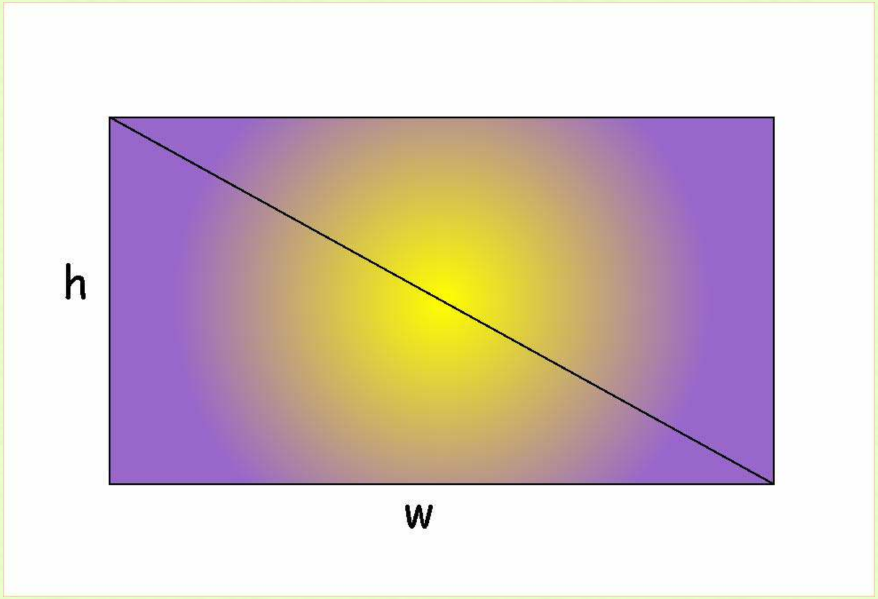
Area

- The area of other polygons are figured out in the same fashion. Let's see if we can deduce the area of a right triangle:



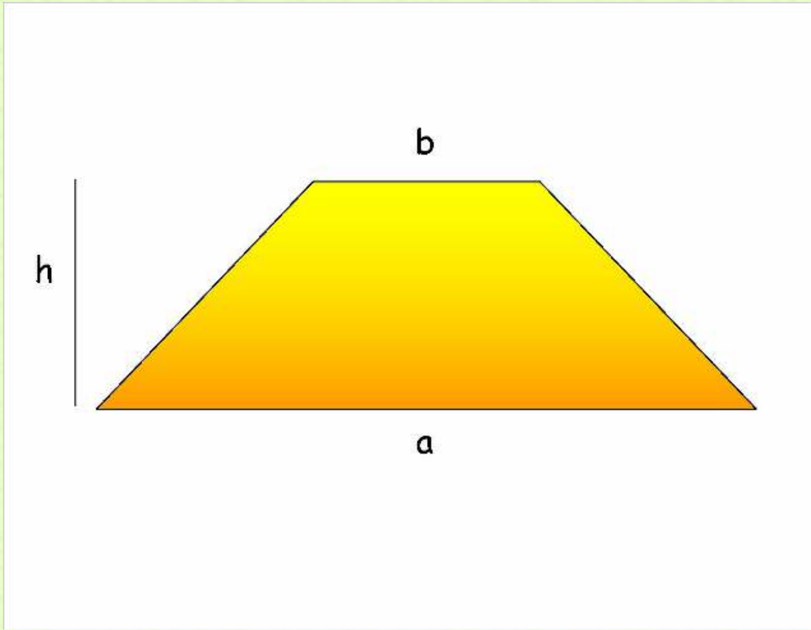
Area

- A (right triangle) = 1/2 w · h

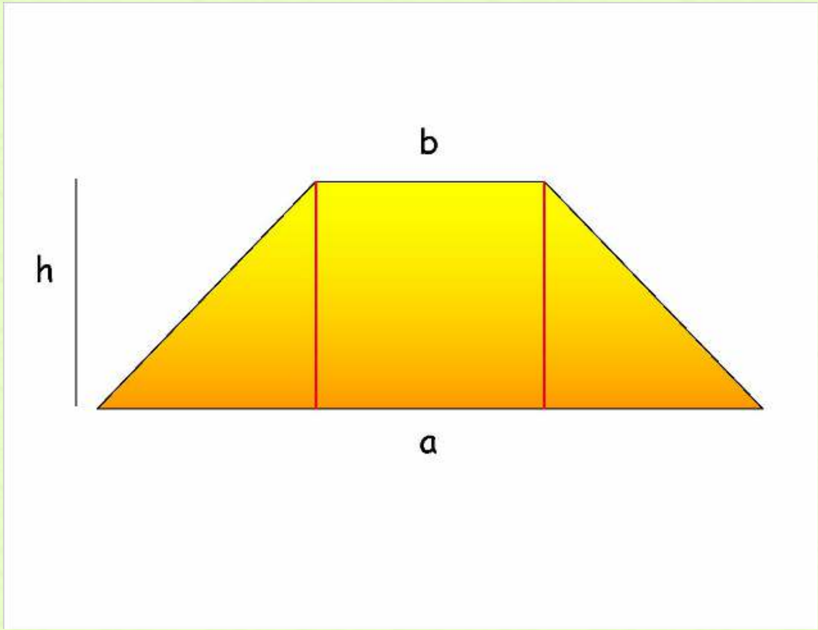


Area

- An isosceles trapezoid:



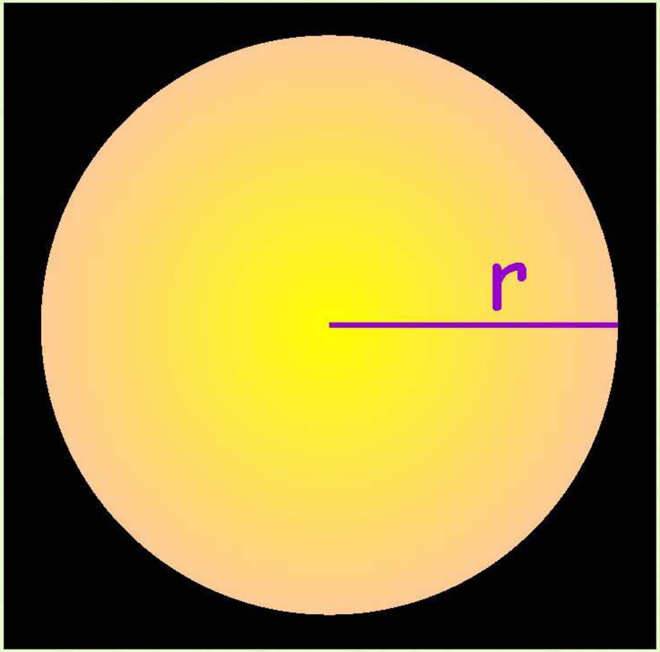
Area



- A (trapezoid) = $h \cdot (a+b)/2$

Area

- The exception to this procedure is the area of a circle; A (circle) = πr^2.



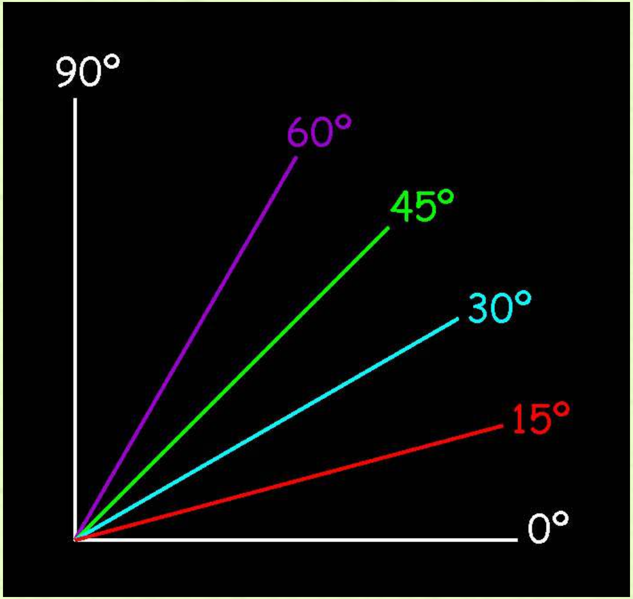
- Most other closed curves need calculus to find their areas.

Angles

- Angles are used to measure a steepness of inclination.

- A 0° angle is considered horizontal,
  while a 90° angle is perpendicular to the horizontal.

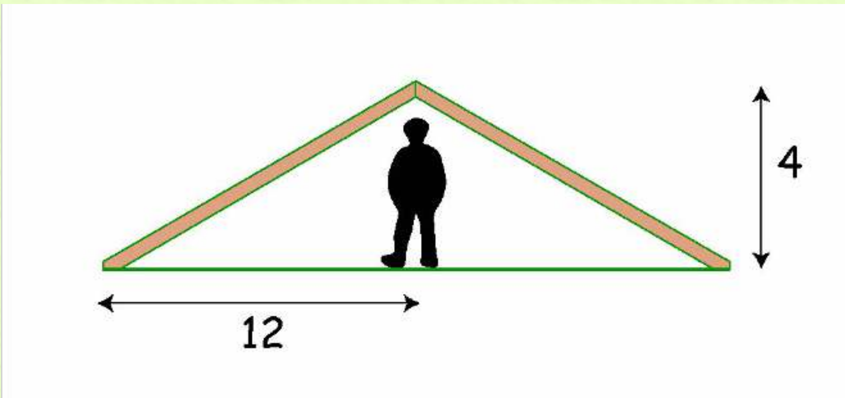
- We'll assume that you've had plenty of practice with angles in previous courses.



Angles

- Some professions use other ways to describe angles.

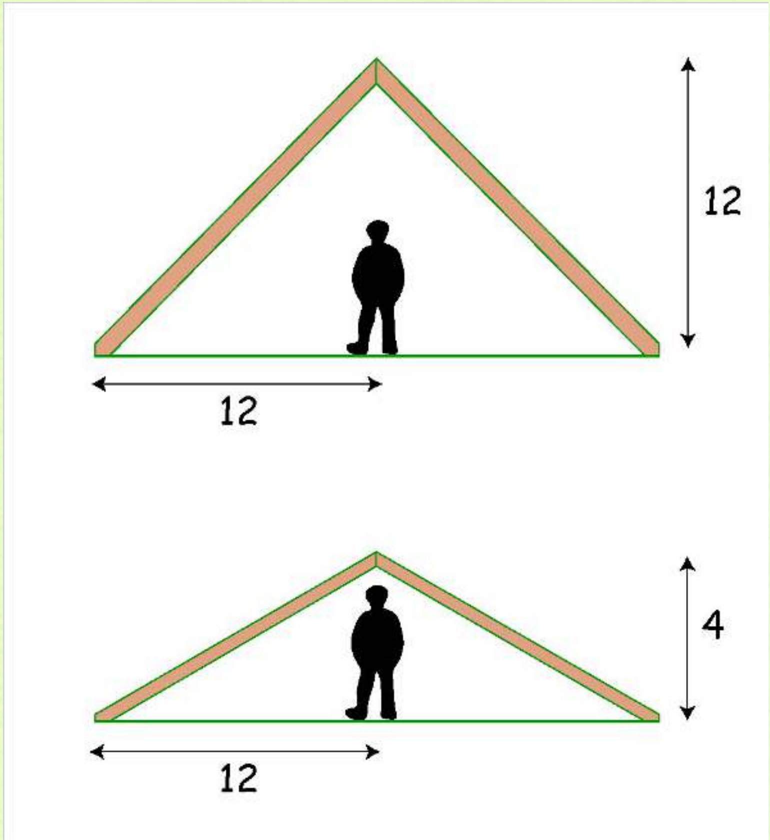
- Builders talk of the pitch of the roof, and describe it, say, as a 4 in 12 pitch, meaning that for every 12 feet horizontal distance, the roof goes up 4 feet.



Angles

- Different roof pitches have different advantages …

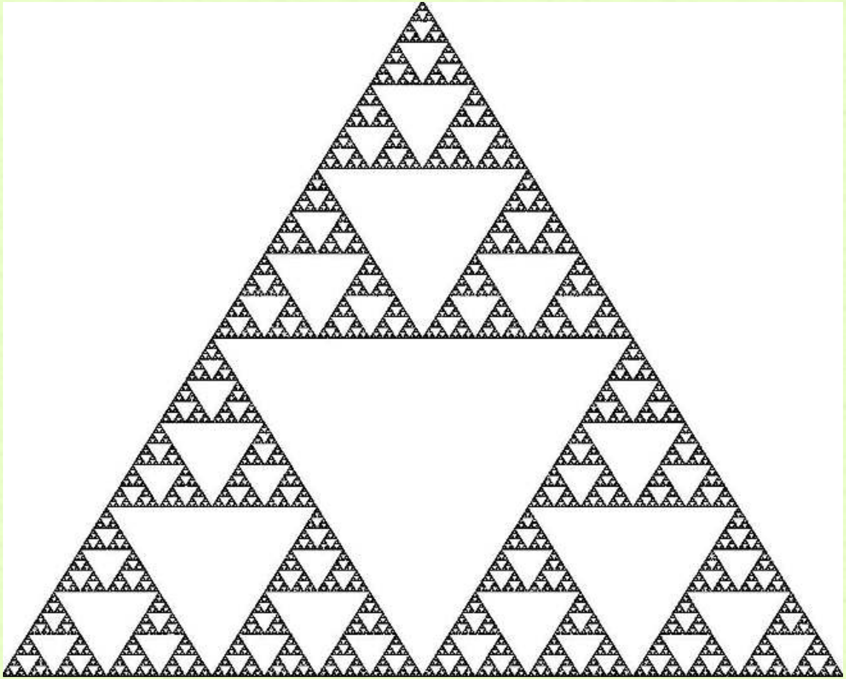
- What angle does a 12 in 12 pitch roof make with the horizontal?



Triangles

- The word “polygon” is derived from the Greek for “many angles”.

- The simplest polygon is the triangle, which has three straight sides.



- Different types of triangles have different names.

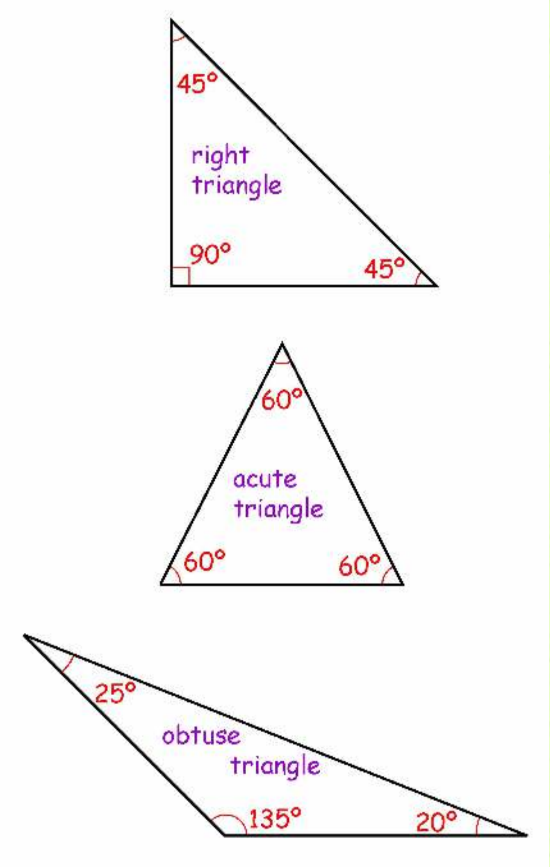
Triangles

- If one considers the angles, every triangle can be classified as one of three types: acute, right, or obtuse.

- An acute angle is less than 90°, and an obtuse angle is more than 90°.

Triangles

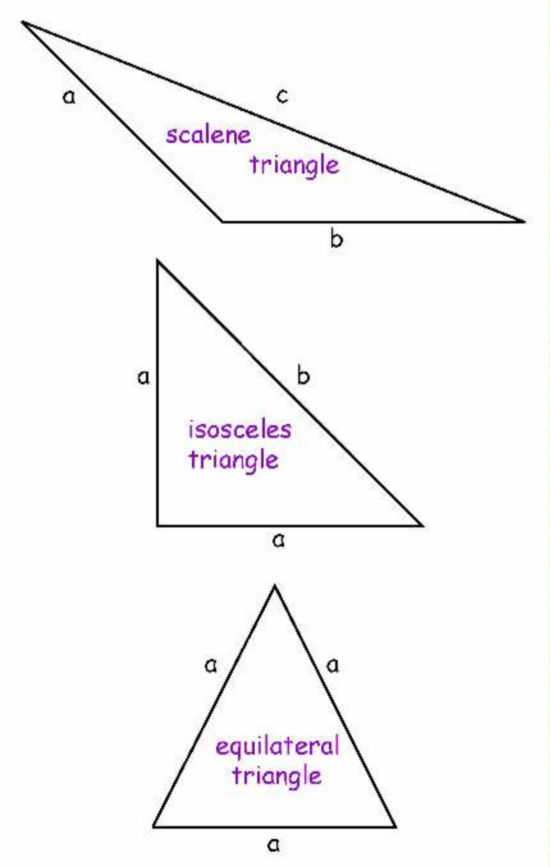
- A right triangle has one angle of 90°, the other two must be acute.
- An acute triangle has three acute angles.
- An obtuse triangle has one obtuse angle and two acute angles.
- Notice something about the sum of the interior angles?



Triangles

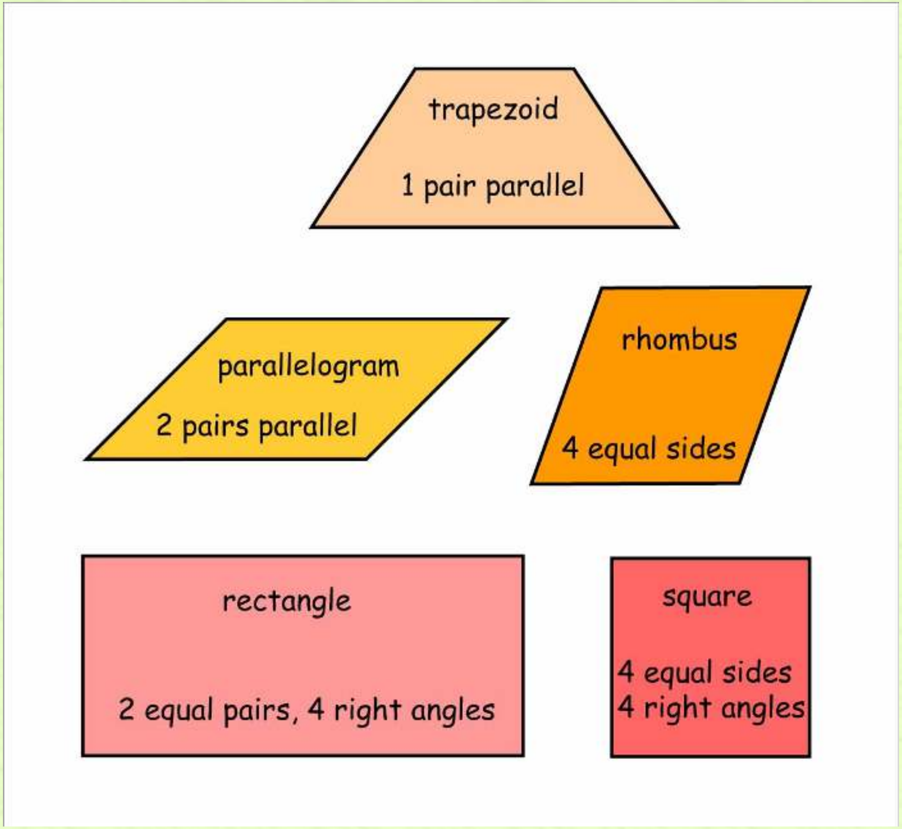
- One may also classify triangles based on the lengths of their sides:
  scalene, isosceles, or equilateral.

- A scalene triangle has three unequal sides.
  An isosceles triangle has two equal sides.
  An equilateral triangle has three equal sides.



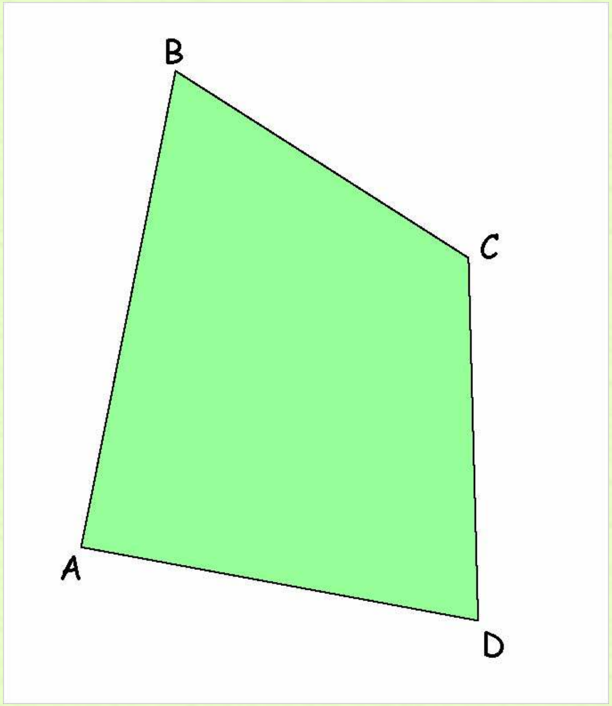
Quadrilaterals

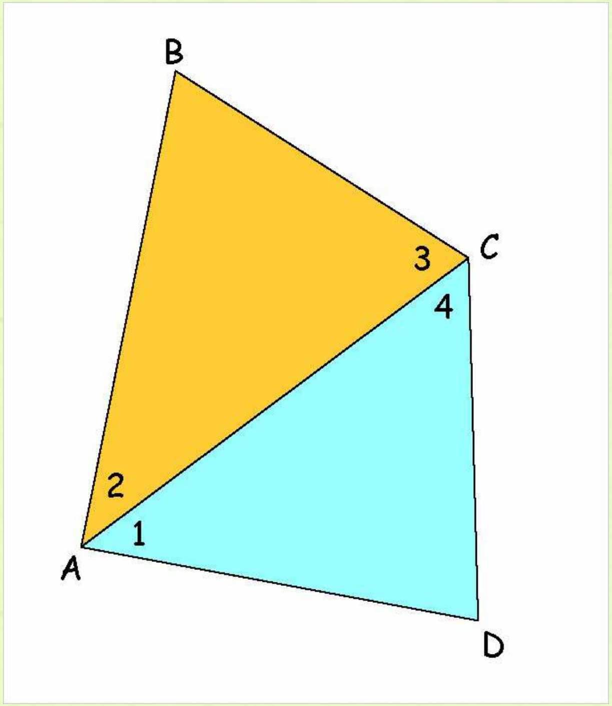
- A quadrilateral is Latin for four sides.
- Again, certain types have names.



Quadrilaterals

- In any quadrilateral a line can be drawn connecting two opposite corners, and dividing the quadrilateral into two triangles.



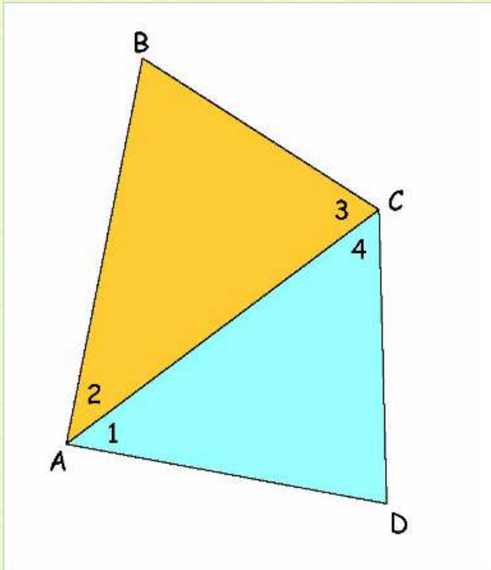


Quadrilaterals

- We figured out earlier that the sum of the angles in a triangle is 180°.

- If we add up the angles in a quadrilateral,

$$A + B + C + D = (1 + 2) + B + (3 + 4) + D$$
$$= (1 + D + 4) + (2 + 3 + B)$$
$$= 180^\circ + 180^\circ$$
$$= 360^\circ$$



Polygons

- The names of the other polygons are taken from the Greek number prefixes followed by -gon.

3=tri	9=ennia	15=pentakaideca
4=terra	10=deca	16=hexakaideca
5=penta	11=hendeca	17=heptakaideca
6=hexa	12=dodeca	18=octakaideca
7=hepta	13=triskaideca	19=enniakaideca
8=octa	14=tetrakaideca	20=icosa

Polygons

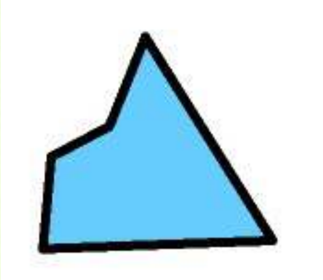
- We found the sum of the angles in a triangle is 180°, and the sum of the angles in a quadrilateral is 360°.

- What do you think the sum of the angles in a pentagon are?

Polygons

- Can you guess what the sum of the angles in an n-sided polygon is?

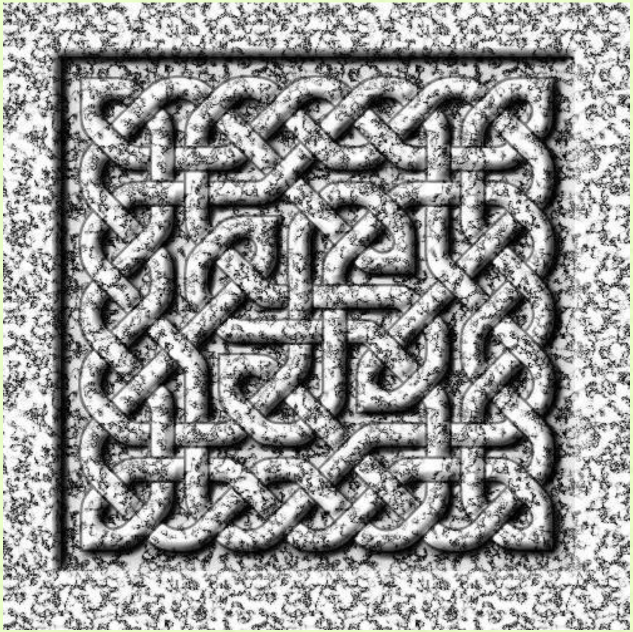
- Sum of angles = $(n - 2) \cdot 180°$

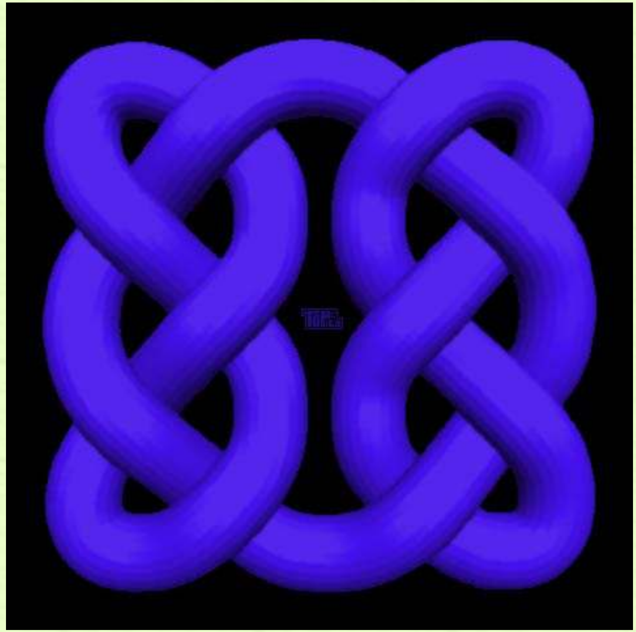


- Every time we add another side to a polygon, a triangle is, in effect, added on to the polygon. So 180° are added to sum of the angles.

Celtic knots

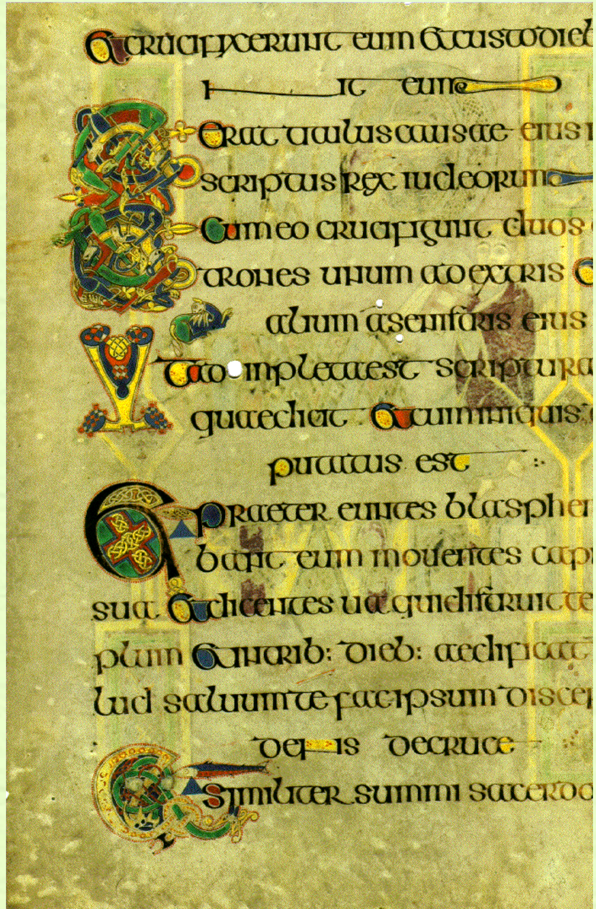
- Ornamental knots appears in Celtic manuscripts, carved stones, and metalwork.





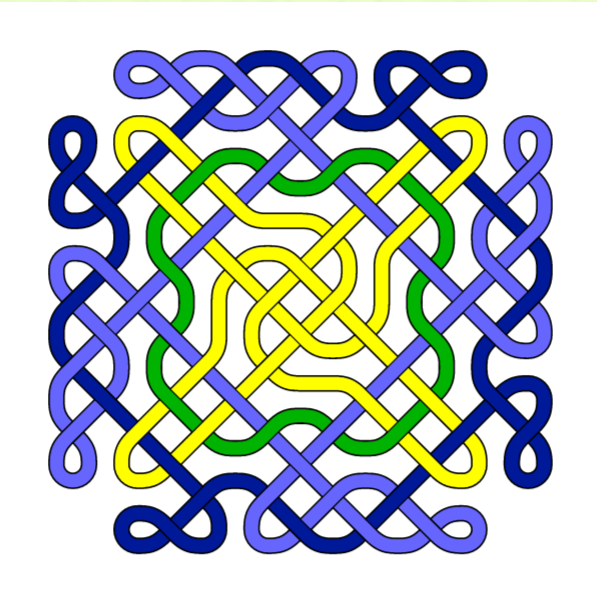
Celtic knots

- The most famous examples occur in manuscripts such as the Lindisfarne Gospels and the Book of Knells.



Celtic knots

- Knotwork exhibits beautiful geometric regularity while offering a great amount of flexibility for the artist.



Celtic knots

- If these look tricky or hard to do … they're not!

- Our first construction will follow section 2.2 of the text, Celtic knots.