

School of the Art Institute of Chicago

Geometry of Art and Nature

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Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

www.counton.org/explorer/morphing/
 O3regularandsemiregulartiling.shtml

www.scienceu.com/geometry/articles/tiling/

mathforum.org/sum95/suzanne/whattess.html

www.mcescher.com/

Class #4

Regular Tilings

Semiregular tilings

Demiregular tilings

Tessellation

• A tessellation or a tiling is a way to cover a floor with shapes so that there is no overlapping or gaps.

 Remember the last jigsaw puzzle piece you put together? Well, that was a tessellation. The shapes were just really weird.



Examples

• Brick walls are tessellations. Each brick's rectangular face is a tile on the wall.

• Chess and checkers are played on a tiling. Each colored square on the board is a tile, and the board is an example of a periodic tiling.





Examples

 Mother nature is a great producer of tilings. The honeycomb of a beehive is a periodic tiling by hexagons.

• Each piece of dried mud in a mudflat is a tile. This tiling doesn't have a regular, repeating pattern. Every tile has a different shape.





• The Alhambra is a Moor palace in Granada, Spain.



• It contains some of today's finest examples of the mathematical art of 13th century Islamic artists.















Tesselmaniac

• Motivated by what he experienced at Alhambra, Maurits Cornelis Escher created many tilings.





• To talk about the differences and similarities of tilings it comes in handy to know some of the terminology and rules.



• In the first class we found that the sum of interior angles in any polygon was

Sum of angles = $(n - 2) \cdot 180^{\circ}$



• We argued that every time we add another side to a polygon, a triangle is, in effect, added on to the polygon. So, 180° are added to sum of the angles.

• If all of the sides have equal lengths, then all the interior angles are the equal. In this special case we call the figure a regular polygon.

 What do you think the interior angle is for a regular polygon with n sides?

nterior angle =
$$\frac{(n-2) \cdot 180^{\circ}}{n}$$

Ir



• Now we can start with the simplest type of tiling, called a regular tiling. It has three rules:

- 1) The tesselation must cover a plane with no gaps or overlaps.
- 2) The tiles must be copies of one regular polygon.
- 3) Each vertex must join another vertex.





1) Tesselations with squares, the regular quadrilateral, can obviously tile a plane.



1)Note what happens at each vertex. The interior angle of each square is 90°. If we sum the angles around a vertex, we get 90° + 90° + 90° + 90° = 360°.







1)The interior angle of every equilateral triangle is 60°. If we sum the angles around a vertex, we get 60° + 60° + 60° + 60° + 60° + 60° = 360° again!

Pentagons

1)Will pentagons work?

• The interior angle of a pentagon is 108°, and 108° + 108° + 108° = 324°.





- Hexagons?
- The interior angle is 120°, and 120° + 120° + 120° = 360°.



Heptagons

- Heptagons? Octagons?
- Not without getting overlaps. In fact, all polygons with more than six sides will overlap.



• So, the only regular polygons that tessellate the plane are triangles, squares and hexagons.



• That was an easy game. Let's make it a bit more rewarding.

Semiregular tiling

• A semiregular tiling has the same game rules except that now we can use more than one type of regular polygon.

 This example is made from a square, hexagon, and a dodecahedron.



Semiregular tiling

 To name a tessellation, wind your way around one vertex counting the number of sides of the polygons that form the vertex.

• Go around the vertex such that the smallest possible numbers appear first.



Semiregular tiling

• Here is another example made from three triangles and two squares.

• There are only 8 semiregular tesselations, and we've now seen two of them: the 4.6.12 and the 3.3.4.3.4

 Your in-class construction will help you find the remaining
 6 semiregular tesselations.



Vertex: 3.3.4.3.4









Vertex: 3.3.3.4.4

Tile Piece





Vertex: 3.3.3.4.4



Tile Piece









Vertex: 3.4.6.4

Tile Piece



Tesselation





Vertex: 3.6.3.6

Tile Piece



Tesselation



Vertex: 3.6.3.6

Tile Piece



Tesselation













Demiregular tiling

• The 3 regular tessellations and the 8 semiregular tessellations we've found are called 1-uniform tilings because all the vertices are identical.

• If the arrangement at each vertex in a tessellation of regular polygons is not the same, then the tessellation is called a demiregular tessellation.

• If there are two different types of vertices, the tiling is called 2-uniform. If there are three different types of vertices, the tiling is called 3-uniform. Examples

• There are 20 different 2-uniform tessellations of regular polygons.



3.4.6.6 / 4.6.12



^{3.3.3.3.3.3 / 3.3.3.4.4 / 3.3.4.3.4}

How many?

• Mathematician Otto Krotenheerdt counted the number of tilings when there are k distinct types of vertices. He found:

k	Tilings
1	11
2	20
3	39
4	33
5	15
6	10
7	7
8 or more	0



3.3.3.4.4 / 3.3.4.3.4 / 3.4.6.4

Unsolved problem

If we allow tilings where vertices can be the same type as long as they are in topologically
different settings, the problem of discovering tilings gets much more complex.

• In fact, counting the total number of 4-uniform tessellations by regular polygons is still an unsolved problem.



"Uh, yeah, Homework Help Line? I need to have you explain the quadratic equation in roughly the amount of time it takes to get a cup of coffee."