Symmetry is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection.

Herman Weyl

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Geometry of Art and Nature

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flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

- nothung.math.uh.edu/~patterns/pdf2000/RayOgar
- www.ucs.mun.ca/~mathed/Geometry/Transformations/frieze.html
- www.joma.org/vol1-2/framecss/rintel/Math/seven.html

Class #6

- Two-Dimensional Symmetries
- Rosette Groups
- Frieze Patterns

Fearful symmetry

- Symmetries are an integral part of nature …

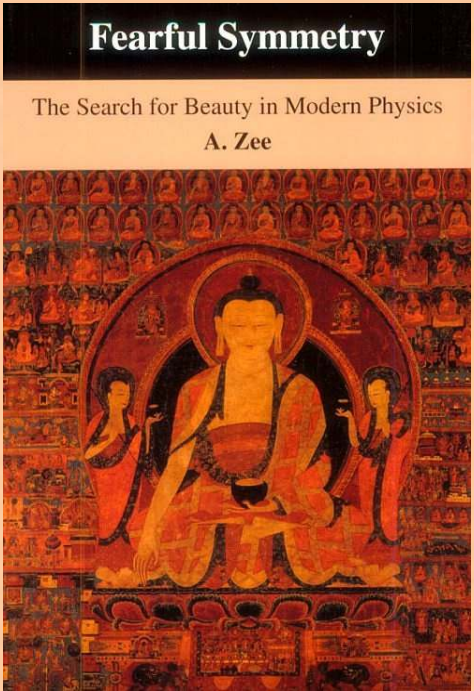
Fearful symmetry

- … and the arts of cultures worldwide.

Hmong textile, Laos

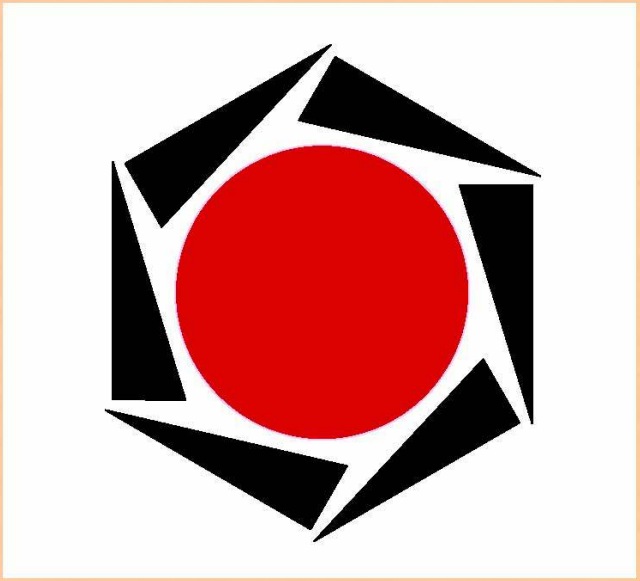
Fearful symmetry

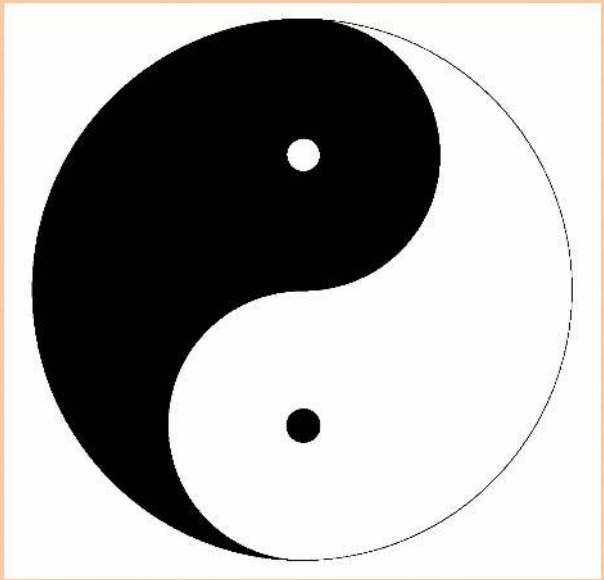
- Symmetry can be found in architecture, crafts, poetry, music, dance, chemistry, painting, physics, sculpture, biology, and mathematics.



Fearful symmetry

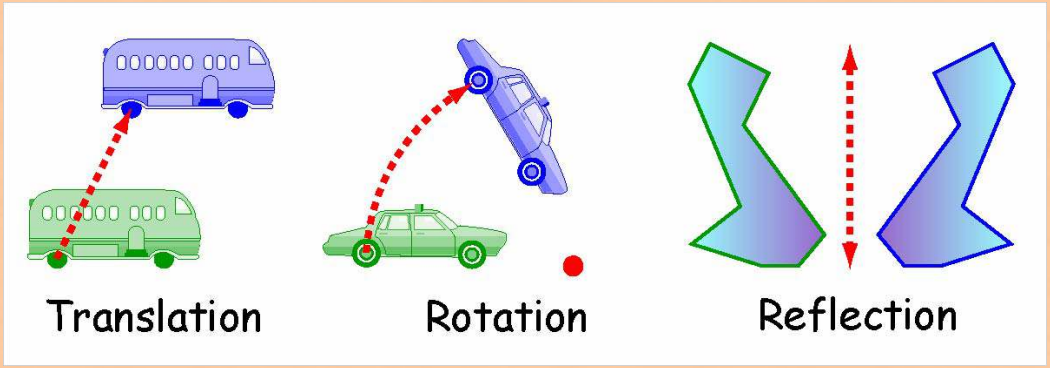
- Because symmetric designs are so naturally pleasing, symmetric symbols are very popular.





Reflection symmetry

- When a figure undergoes an isometry and the resulting image coincides with the original, then the figure is symmetrical. Different isometries yield different types of symmetry.



- If a figure can be reflected over a line in such a way that the resulting image coincides with the original, then the figure has reflection symmetry.

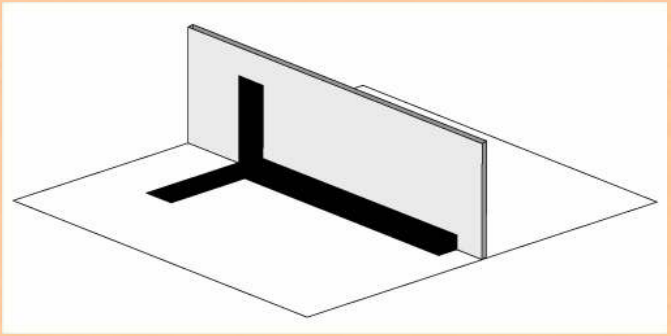
Reflection symmetry

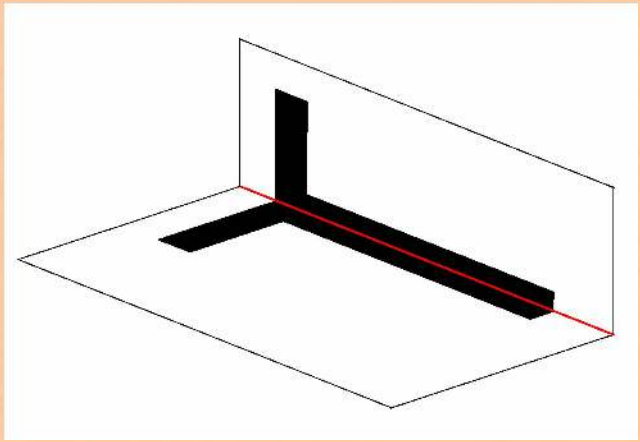
- Reflection symmetry is also called line symmetry or bilateral symmetry or mirror symmetry.
  The reflection line is called the line of symmetry.

- This Navajo rug has two lines of symmetry.

Reflection symmetry

- The letter T, when reflected about its line of symmetry with a mirror, is identical to the T in the original position.





- You can test a figure for reflection symmetry by tracing and folding it. If you can fold it so that one half exactly coincides with the other half, the figure has reflection symmetry.

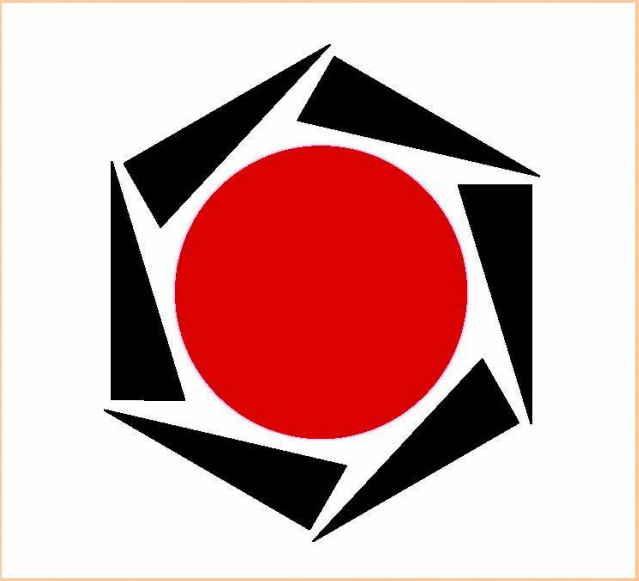
Reflection symmetry

- How many lines of symmetry do the butterfly, leaves, and Hmong textile have?

Rotational symmetry

- If a figure can be rotated about a point in such a way that its rotated image coincides with the original figure, then the figure has rotational symmetry.

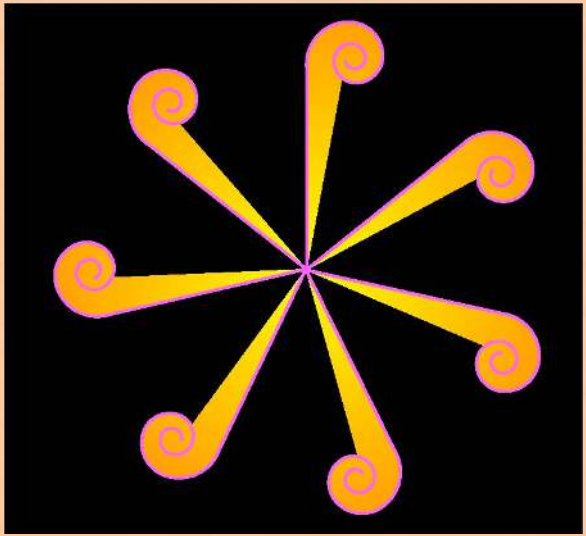
- This logo design, for example, has a six-fold rotational symmetry.



Rotational symmetry

- You can trace a figure and test it for rotational symmetry. Place the copy over the original and rotate the copy about the suspected symmetry point.

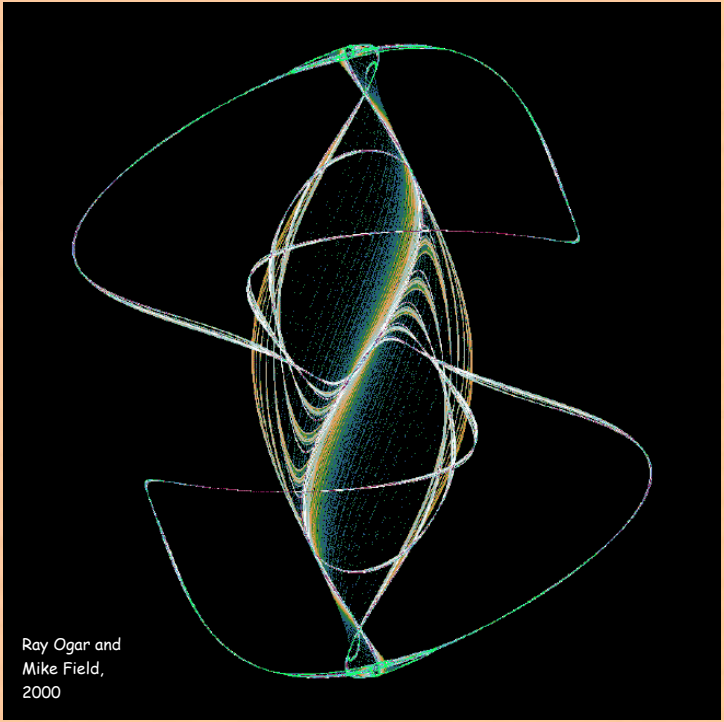
- Count the number of times the copy and the original coincide with the copy until it is back in the position it started in.



What is the n-fold symmetry of this logo?

Rotational symmetry

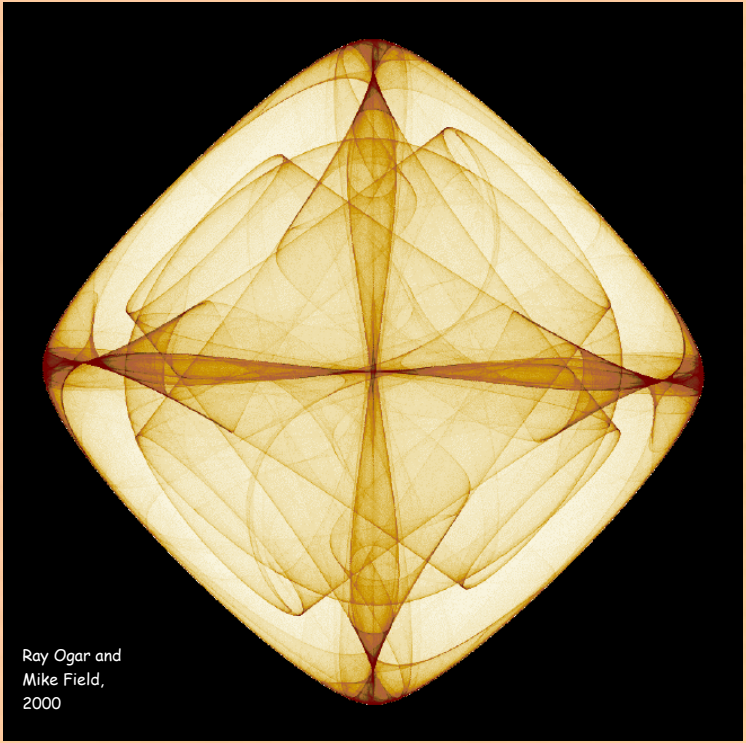
- What is the n-fold rotation symmetry of this design?



Ray Ogar and
Mike Field,
2000

Rotational symmetry

- Many designs, like this logo, have both reflection and rotational symmetry.

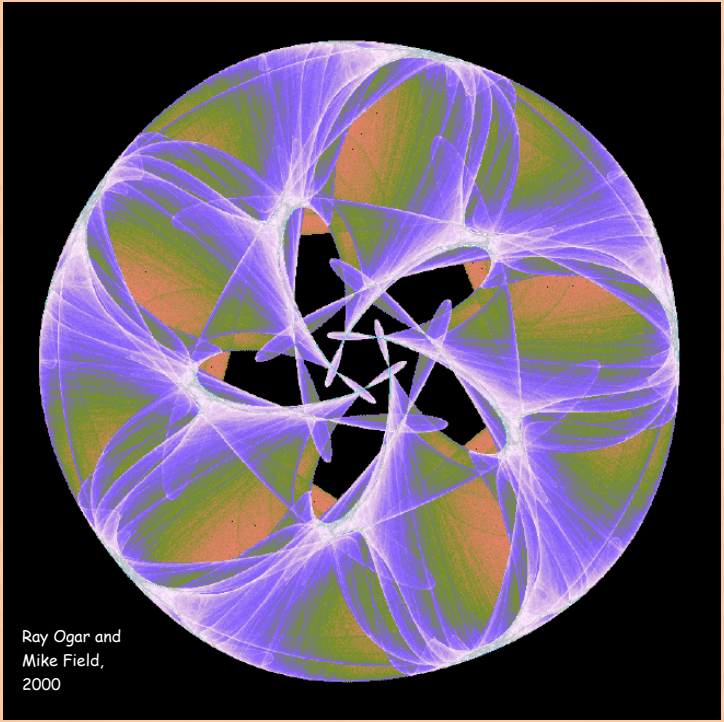


Ray Ogar and Mike Field, 2000

How many reflection symmetries are there? How many rotational?

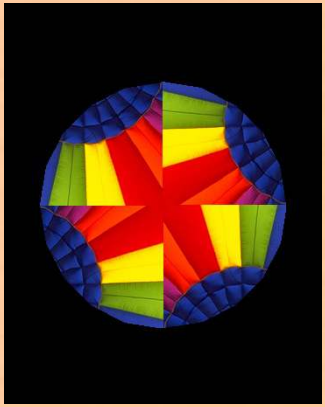
Rotational symmetry

- Other designs only have rotational symmetry.



Rosette groups

- Things with rotational symmetry about a single point and no reflection symmetries belong to the cyclic rosette group, written C_n.



- Things with rotational symmetry about a single point and reflection symmetries about a line belong to the dihedral rosette group, written D_n.

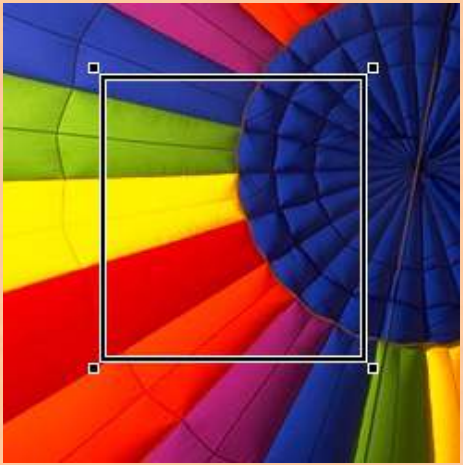
Rosette groups

- These two rosette groups are the only possible ones for things with rotational symmetry, a mathematical theorem first proved by Leonado da Vinci.

- But, there are an infinite number of cyclic C_n and dihedral D_n figures!

Cyclic rosette group

- Here is an example of C_1 through C_{12} :



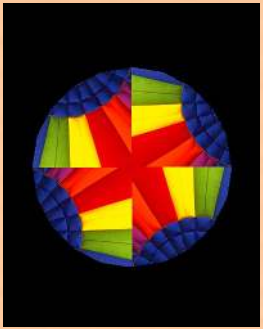
Source box

C1

C2

Cyclic rosette group

C3



C4

C5

C6

C7

C8

C9

C10

C11

C12

Dihedral rosette group

- Here is an example of D_1 through D_{12} :

Source box

D1

D2

Dihedral rosette group

D3

D4

D5

D6

D7

D8

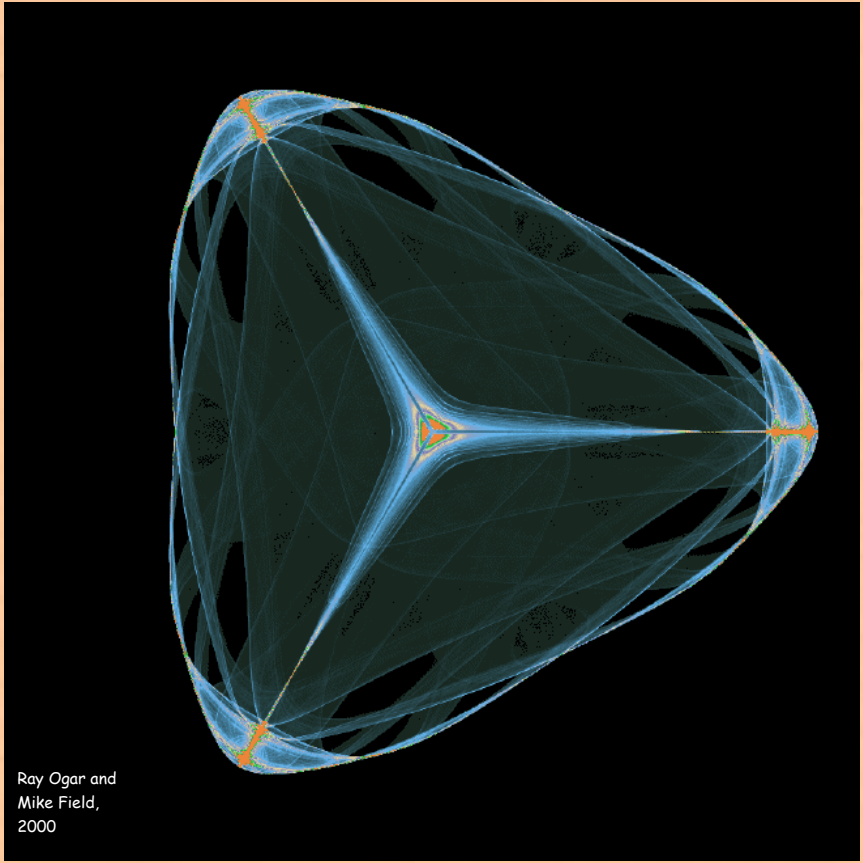
D9

D10

D11

D12

Name that rosette group



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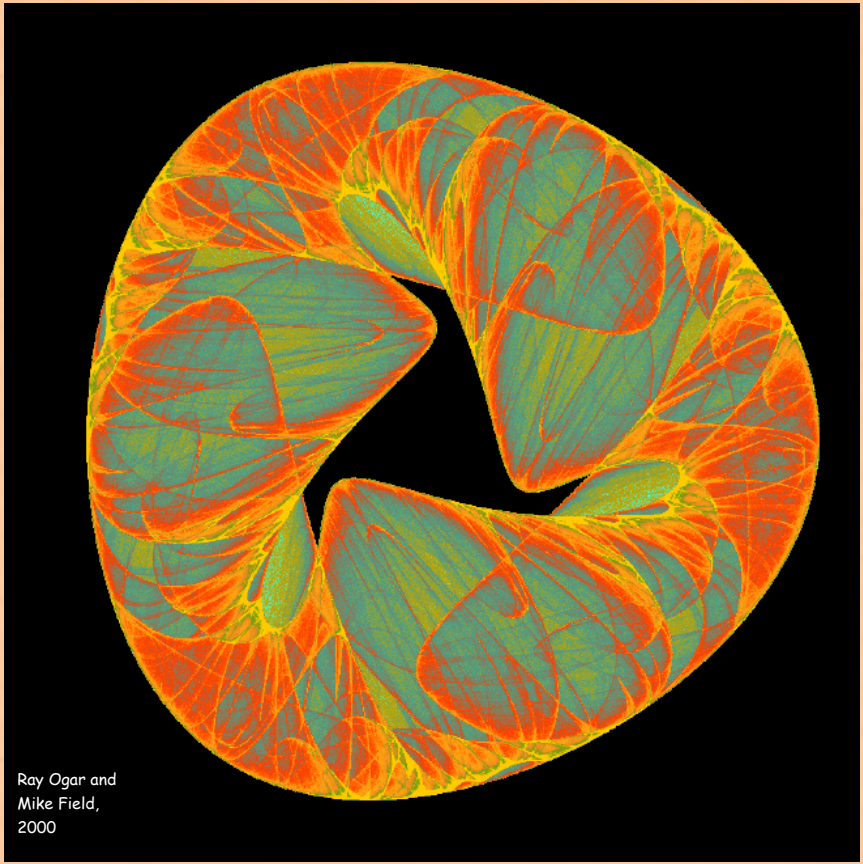
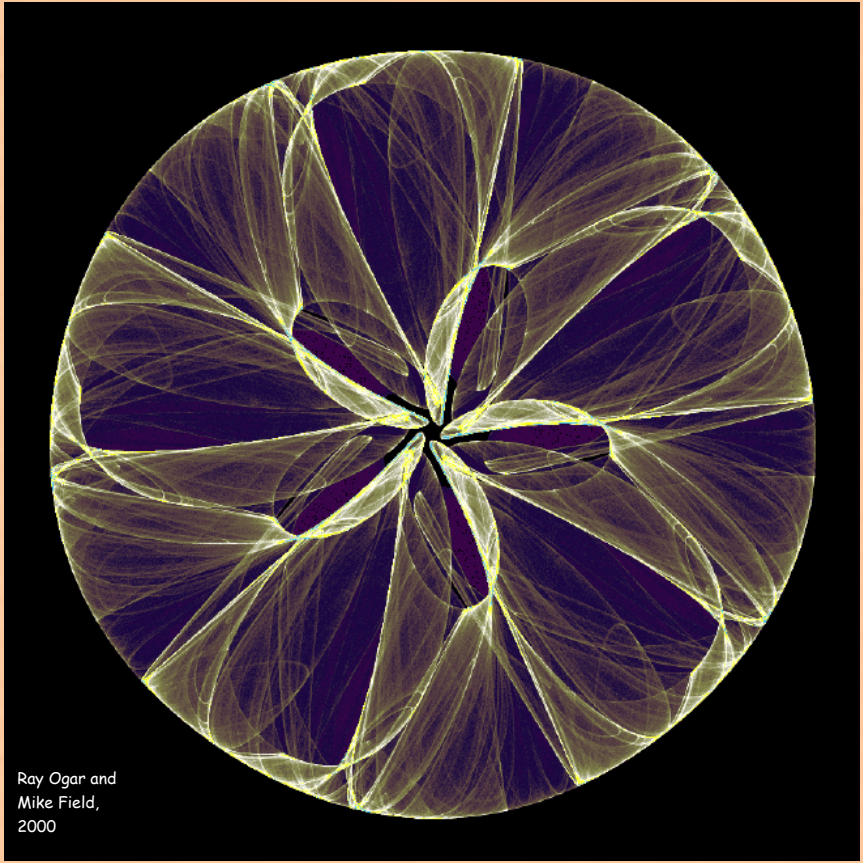
D3

Name that rosette group



C3

Name that rosette group



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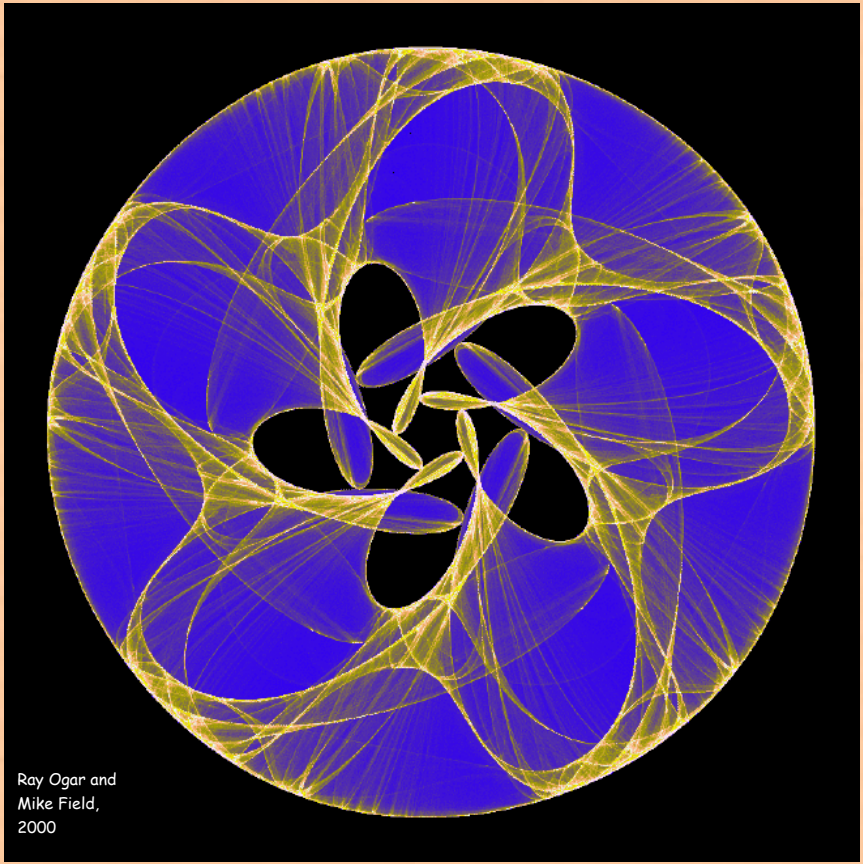
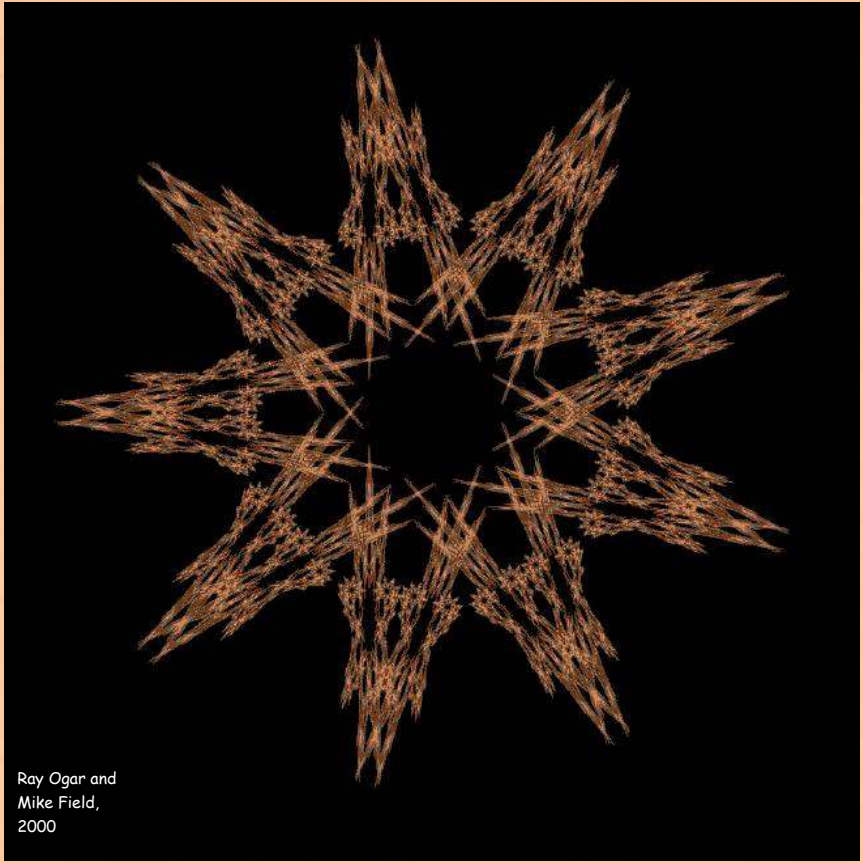
C5

Name that rosette group



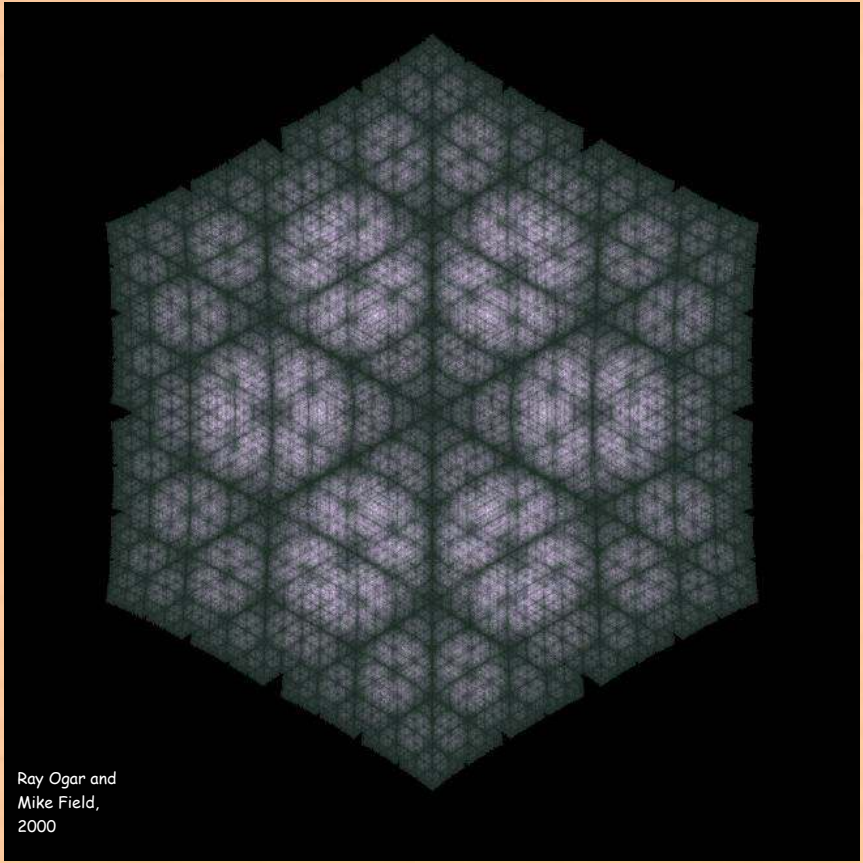
C5

Name that rosette group



D9

Name that rosette group



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D6

Frieze Patterns

- Frieze, or border, patterns are formed by repetitions of a motif along a line.

- There are only seven types of frieze patterns!
  Contrast this with the infinite number of cyclic and dihedral rosette groups.

Frieze Patterns

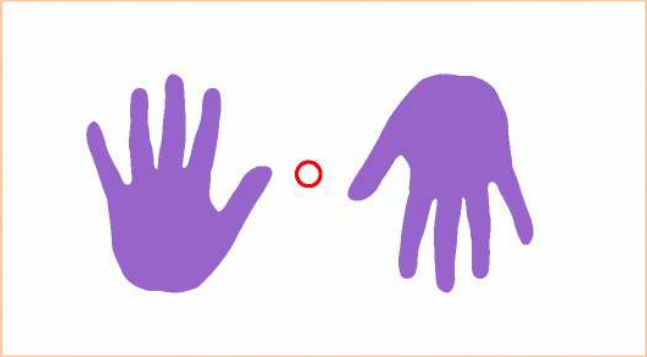
- Every human culture (even people living in caves) that has left artifacts has created line designs based on only seven types of border patterns.

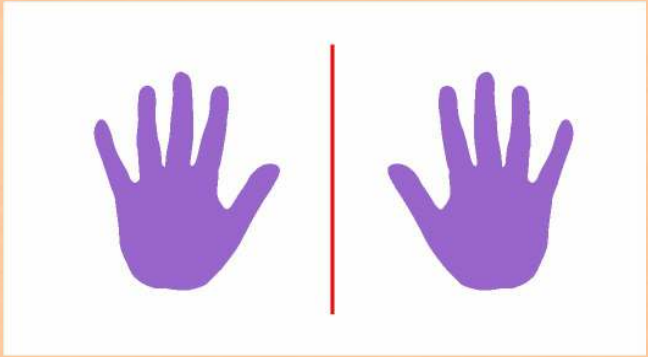
Frieze Patterns

- That being so, one might assume they would have standard names by now, but such is not the case. You'll pick up two of these notations when you play with frieze patterns in today's in-class construction.

Frieze Patterns

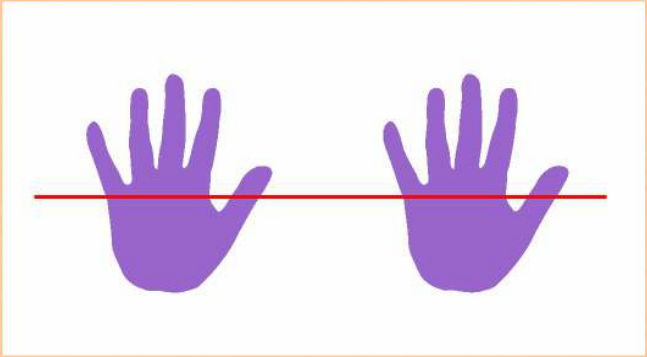
- Besides rotation and reflection symmetries …

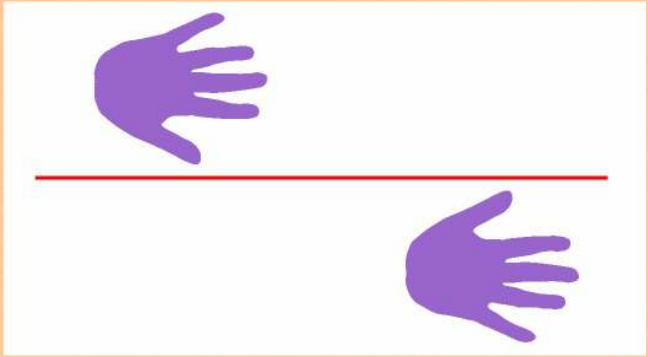




Frieze Patterns

- … we can also have translation and glide reflection symmetries along a line.





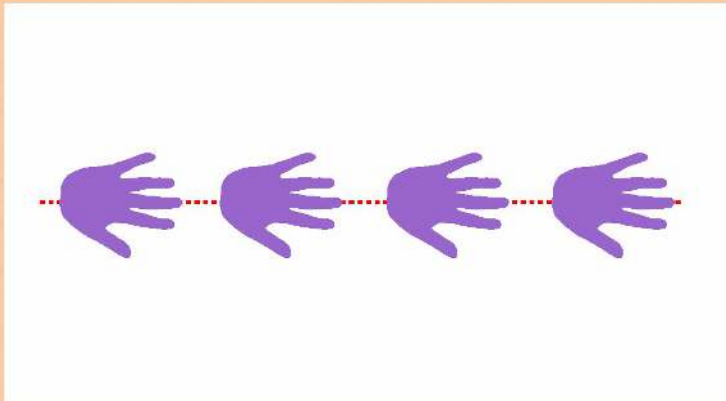
Frieze Patterns

- Rotation, reflection, translation, and glide reflection are the only four isometries on the plane; the only four transformations that preserve size and shape without distortion.

- From these four isometries, there are only seven types of frieze patterns. Let's look at the seven types.

Hop

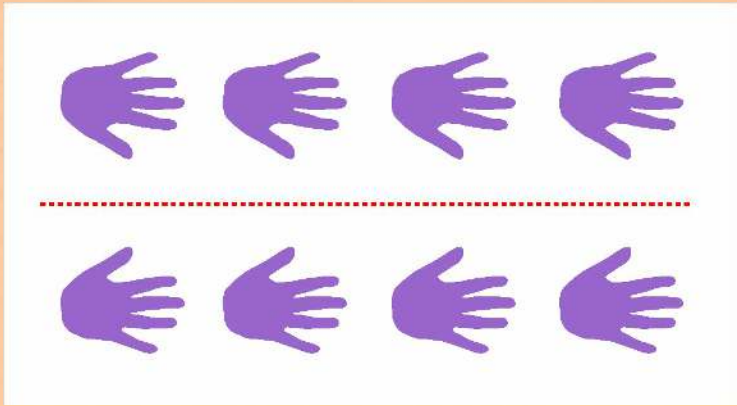
- If we only apply translations to a motif, we get the frieze pattern called a hop.



Mosaic Border, Alcázar de los Reyes Cristianos, Córdoba, Spain

Jump

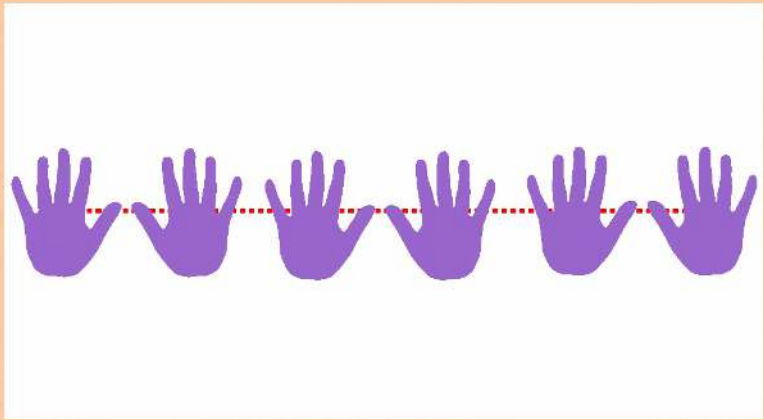
- Reflecting a motif across the center gives us two hands, a left and a right.
  We can then translate this doubled motif along the paper.
  We'll call this frieze pattern a jump:



Ceiling, Mezquita Córdoba, Spain

Sidestep

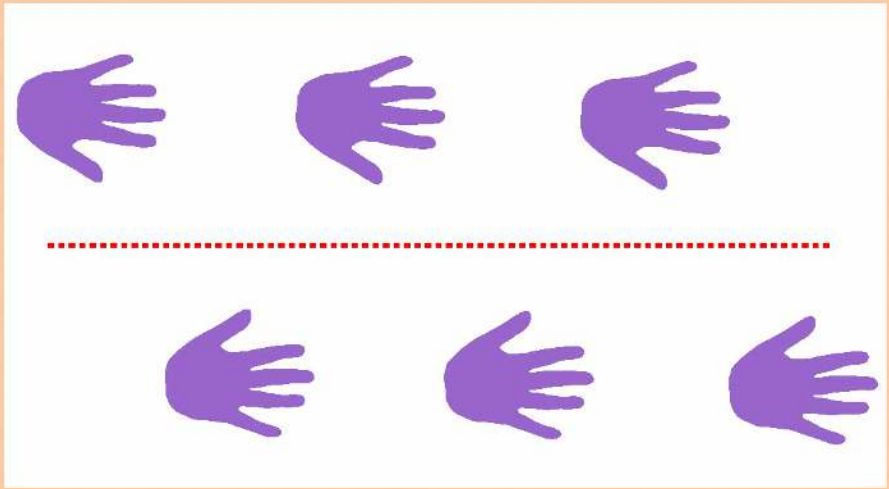
- Reflecting a motif across a vertical line perpendicular to the dotted line, and then translating the doubled motif, we get the sidestep frieze pattern.



Tile Frieze, Palacio de Velázquez, Parque de Retiro, Madrid, Spain

Step

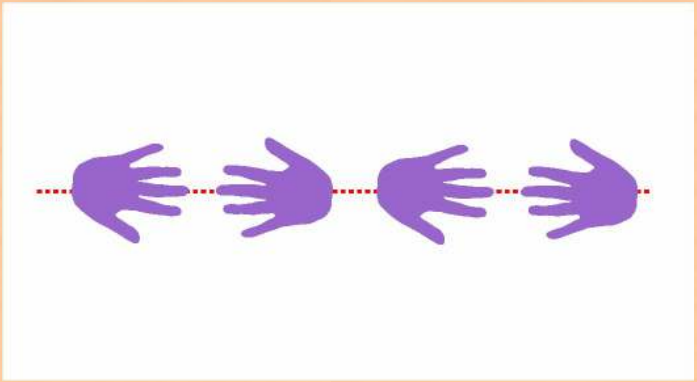
- A simple glide reflection along the center gives the frieze pattern called a step:



- So far we've only used translations, reflections, and glide reflections. Yet we've found four of the frieze patterns. The last three patterns involve rotations.

Spinning hop

- Rotating a motif by 180° and then translating gives a spinning hop:

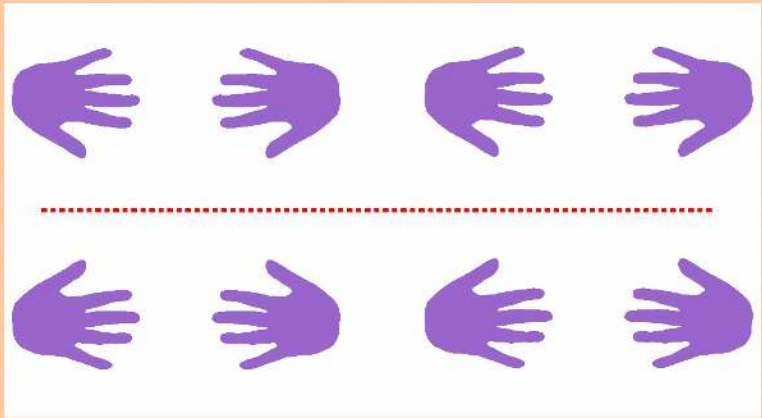


Only left hands!

Meander Frieze, San Giorgio Maggiore,Venice, Italy

Spinning jump

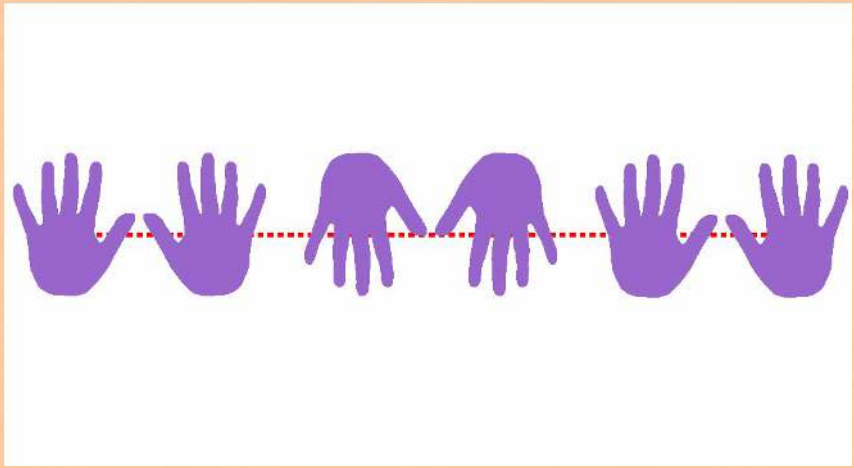
- A spinning jump is generated by reflecting the motif across the center line, rotating the doubles motif by 180°, and translating:



Back of a Bench, Baños de la María de Padilla, Reales Alcázares, Seville, Spain

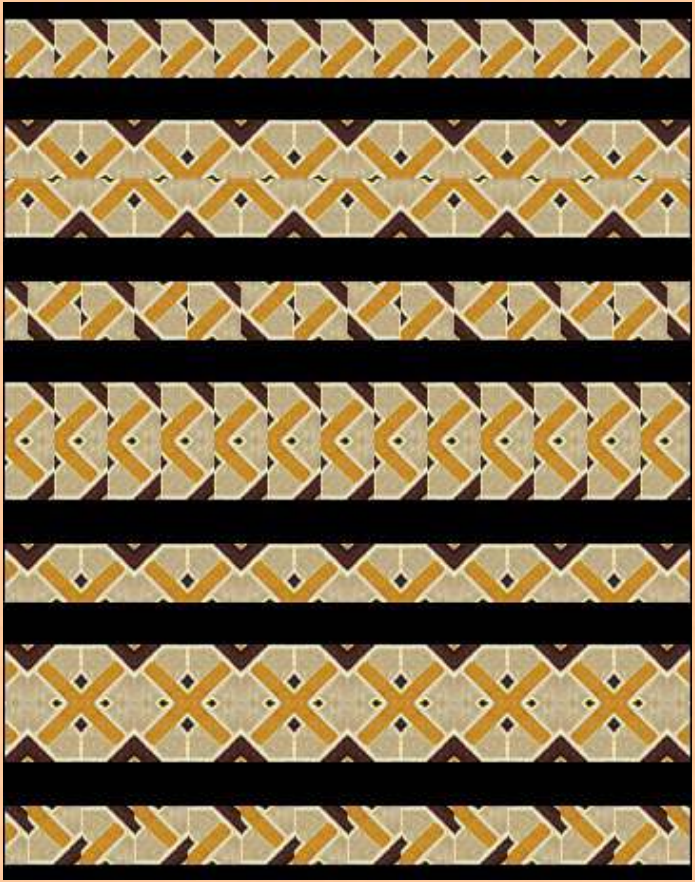
Spinning sidestep

- A spinning sidestep reflects a motif across a line perpendicular to the center line, rotating the doubled motif by 180°, and translating:



Mosaic, Nuestra Señora de la Almundena, Madrid, Spain

Source box



Hop

Spin side

Spin hop

Jump

Side step

Spin jump

Step

Playtime

- You'll create some frieze patterns during our in-class construction today.

- As you go about your way,
  see if you can identify the rosette groups and
  frieze patterns that you find.