Research is what I am doing when
I don't know what I'm doing.

Wernher von Braun

School of the Art Institute of Chicago

Geometry of Art and Nature

Frank Timmes

ftimmes@artic.edu

flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

- home.connexus.net.au/~robandfi/Stella.html
- www.georgehart.com/virtual-polyhedra/zometool.html
- www.dartmouth.edu/~matc/math5.geometry/unit6/unit6.html

Class #8

- Polyhedrons
- Pyramids, Prisms & Antiprisms
- Platonic Solids

Solids

- Solids have three dimensions: length, width, and height.

- You see and touch solids all the time.
  Some solids, like rocks and plants, are very irregular.

ruby

emerald

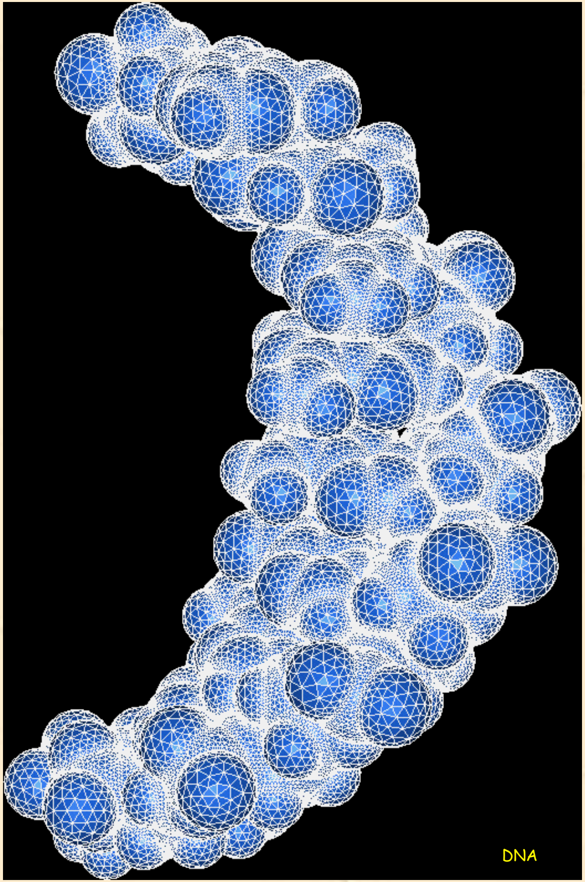
Solids

- But many solid objects have shapes that can easily be described using common geometric terms.

- Some of these occur in nature: viruses, oranges, crystals, the earth itself. Others are manufactured: books, buildings, baseballs, and ice cream cones.

Solids

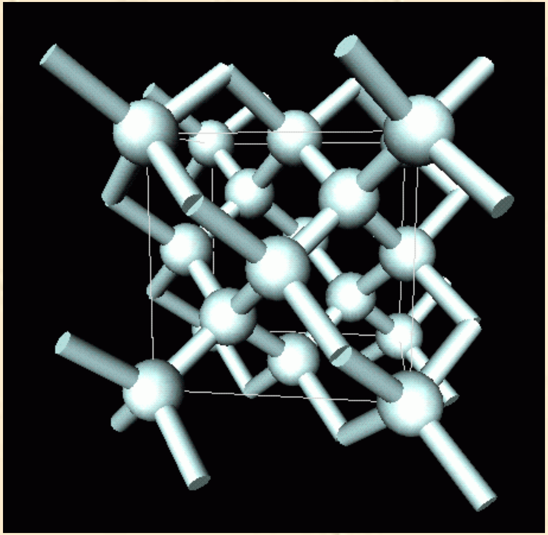
- At the molecular level, three-dimensional geometry plays a critical role in a number of common substances.

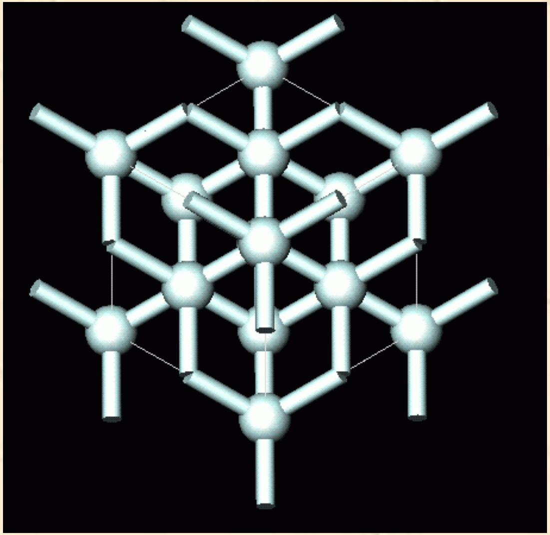


Carbon

- The three-dimensional structure of carbon is an interesting example.

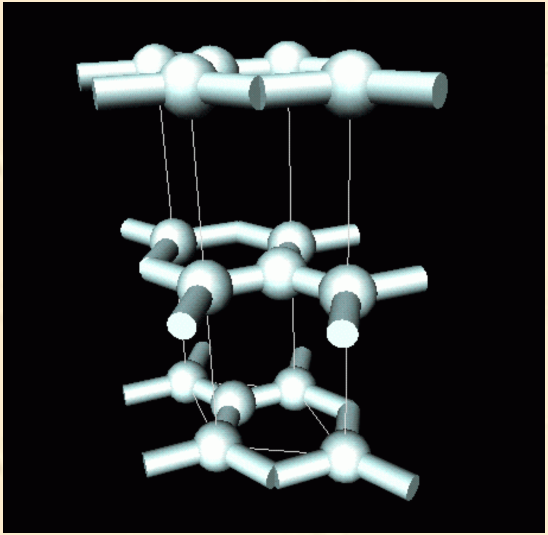
- Carbon atoms arranged in a tetrahedral lattice result in diamonds, one of the hardest materials around!

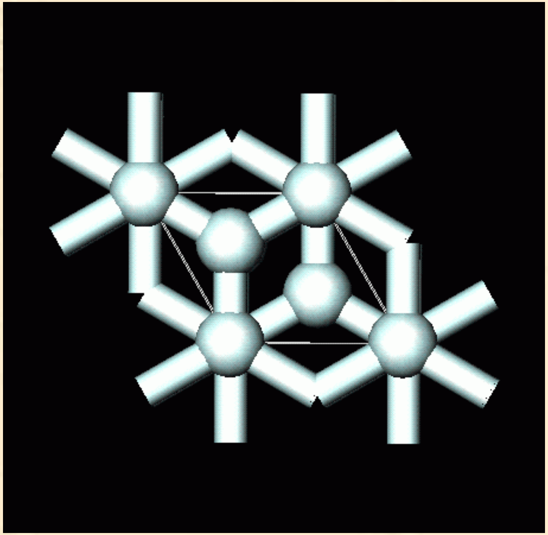




Carbon

- But, carbon bonded in planes of hexagonal rings results in graphite, a soft material used as a lubricant.

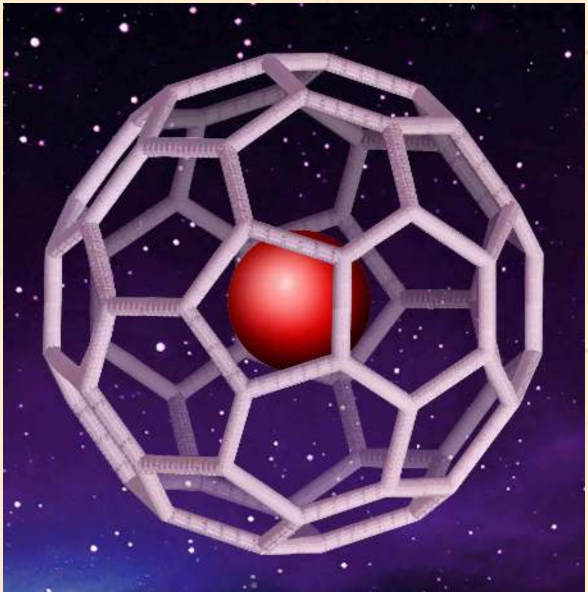




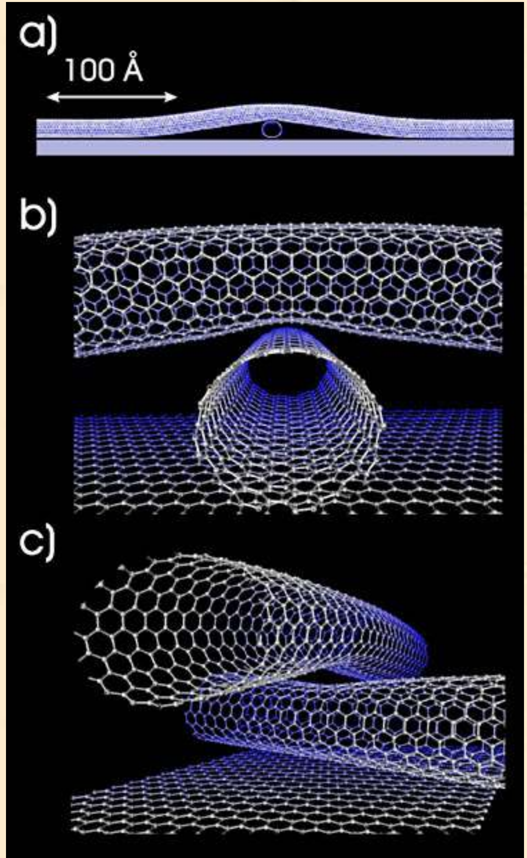
- The results are totally different with the same chemical element. The difference is in the geometry.

Carbon

- We've recently discovered that carbon can also bond into very large, symmetrical molecules called fullerenes.

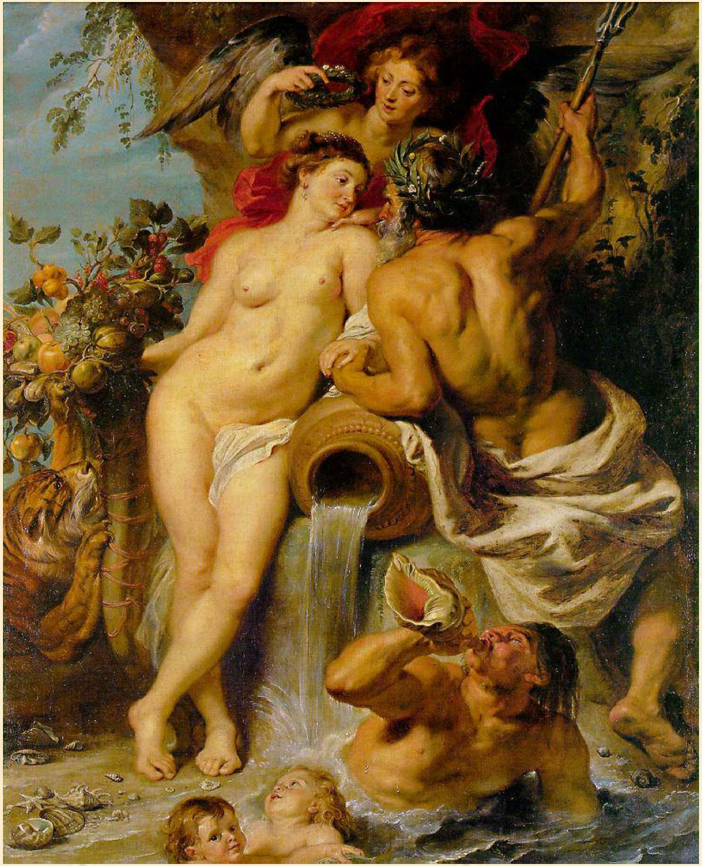


Buckyballs and nanotubes



Water

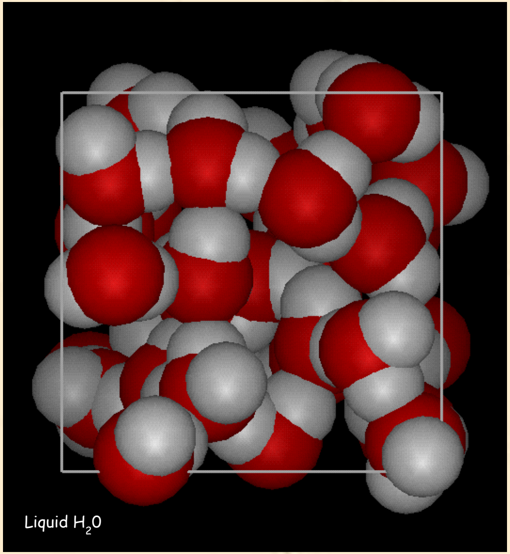
- Three-dimensional geometry plays one of its most important roles at the molecular level with simple H_2O, or water.

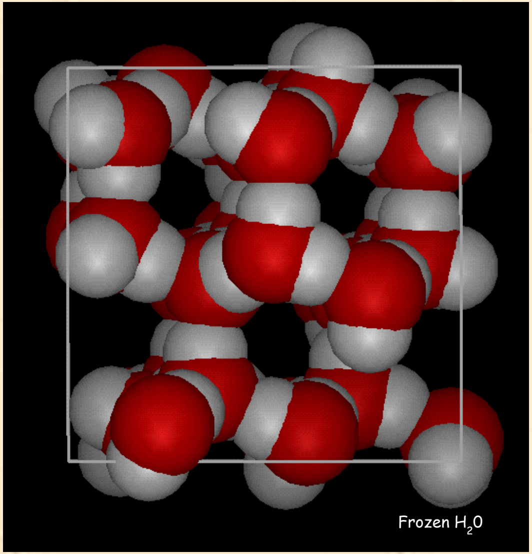


The Union of Earth and Water
1618, Peter Paul Rubens
Oil on canvas
222.5 x 180.5 cm,
Hermitage, St. Petersburg

Water

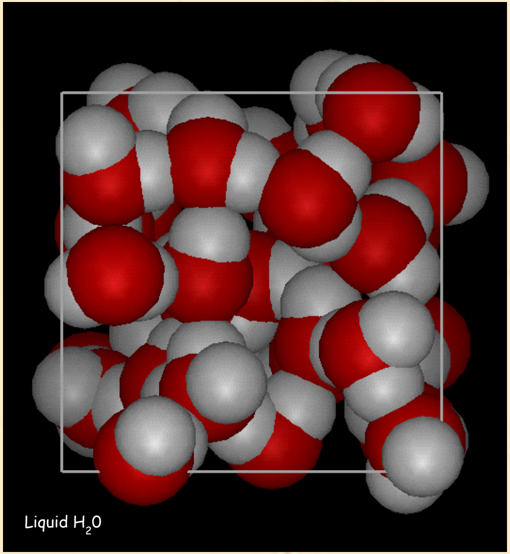
- Nearly all substances shrink in volume (become more dense) when they decrease in temperature. Water is a rare exception.
  Water increases in volume when frozen, becoming less dense.

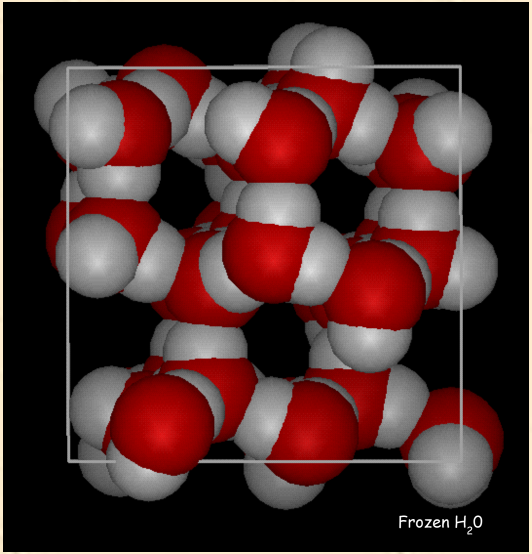




Water

- The motion of molecules slows down as the temperature decreases. But rather than packing closer together, as in most substances, the molecules in water move apart into a rigid three-dimensional lattice.



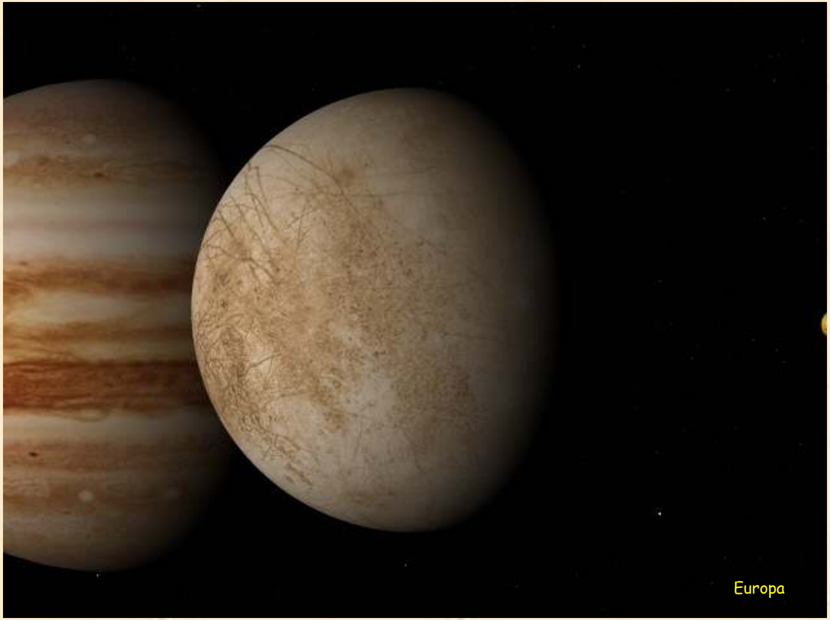


Water

- Water's exceptional behavior is very important for life on this planet. Because ice is less dense than water, ice floats on water, staying closer to the sun's heat.

Water

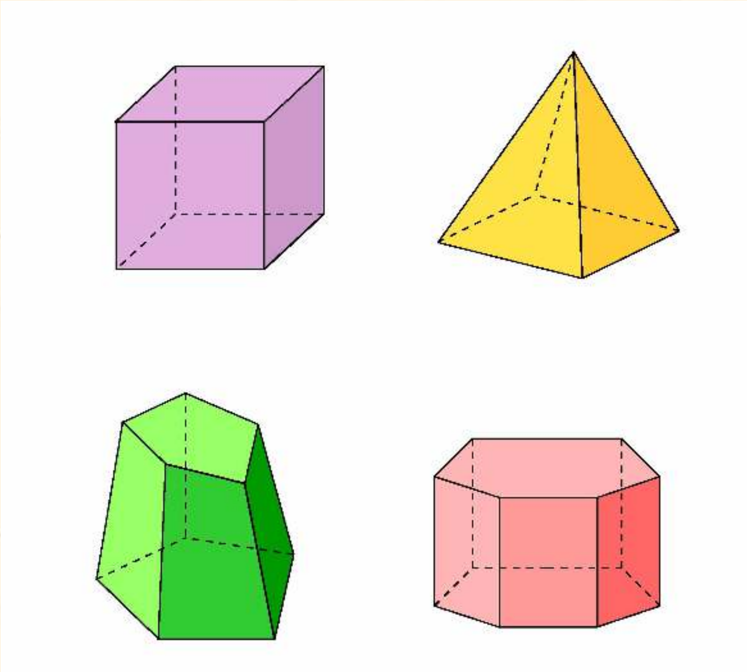
- If ice were denser than water, it would sink to the bottom of the ocean, away from heat sources. Eventually the oceans would fill from the bottom up with ice, and we would have an ice planet.



Polyhedra

- We'll start with a group of solids called polyhedra.

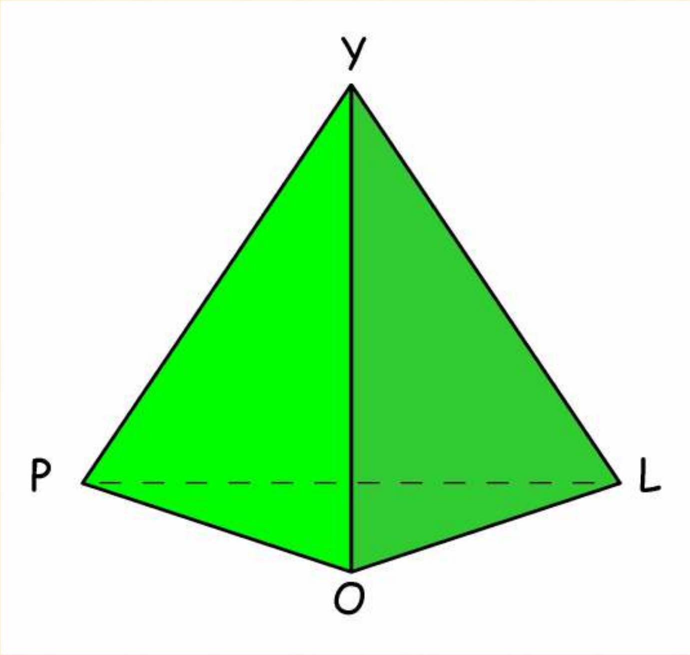
- A polyhedron is a three-dimensional figure whose faces, or sides, are polygons.



Polyhedra

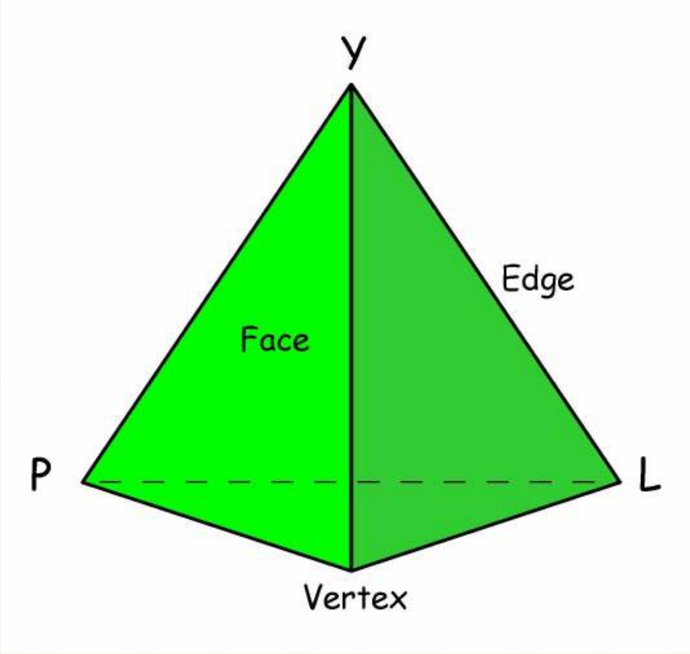
- Triangle POY is one of four triangular faces in this polyhedron.
  What are the other three?

- A segment where two faces intersect is called an edge.
  Segment PO is one of the six edges.
  Name the others.



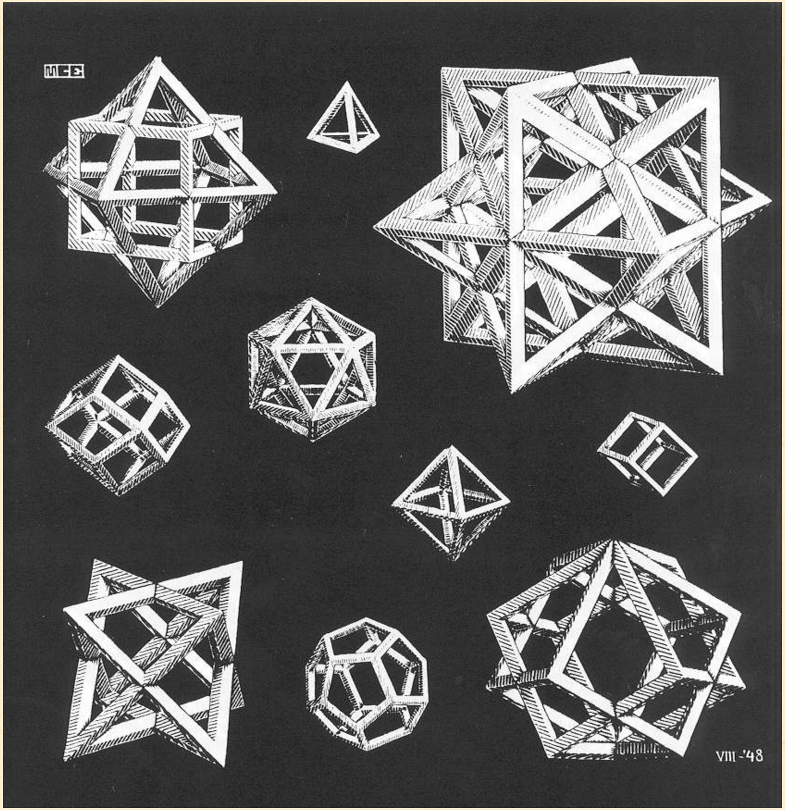
Polyhedra

- A point of three or more edges is called a vertex.
- Points P, O, L and Y are the four vertices.



Polyhedra

- Just as a polygon is classified by its number of sides, a polyhedron is classified by its number of faces.



Polyhedra, 1948, M.C. Escher

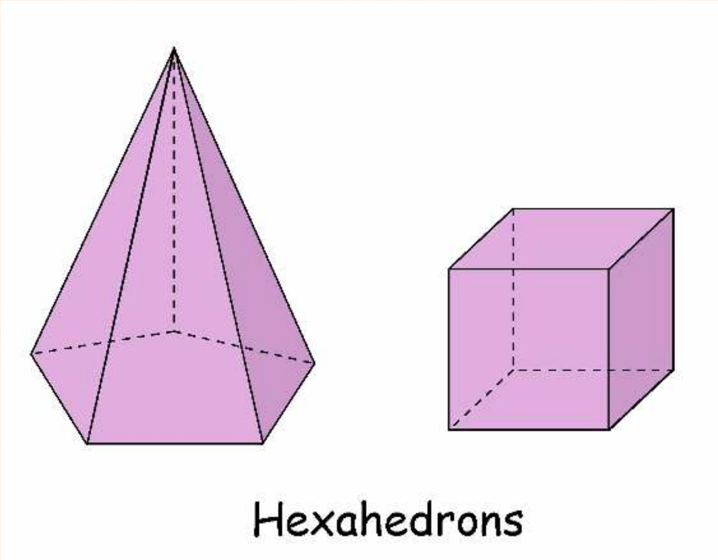
Polyhedra

- The prefixes for polyhedra are the same as they are for polygons, except quadrilaterals. A polyhedron with four faces is called a tetrahedron.

3=tri	9=ennia	15=pentakaideca
4=terra	10=deca	16=hexakaideca
5=penta	11=hendeca	17=heptakaideca
6=hexa	12=dodeca	18=octakaideca
7=hepta	13=triskaideca	19=enniakaideca
8=octa	14=tetrakaideca	20=icosa

Polyhedra

- The prefix penta- means "five", so a pentahedron is a polyhedron with 5 faces.



- How would you classify a polyhedron with eight faces?

Pyramids

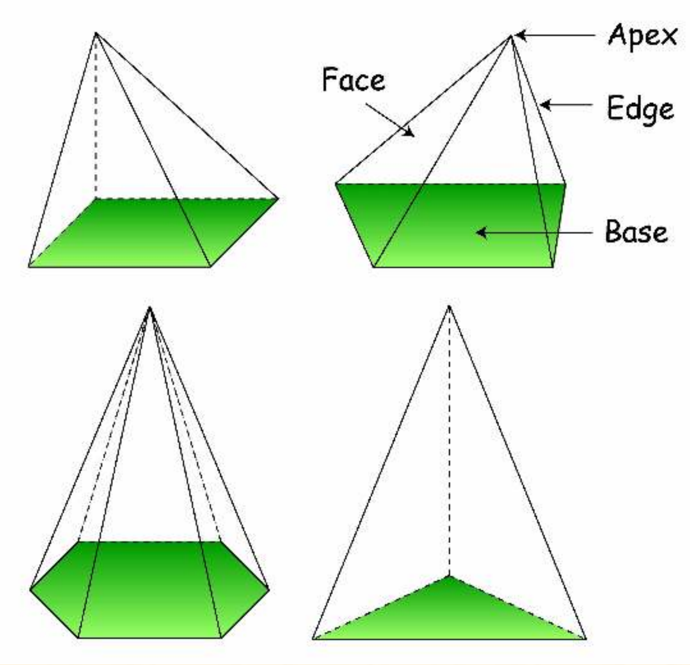
- There are many types of polyhedra. We'll explore some of these over the next couple of classes. Let's start with pyramids.

A Robot visits the Pyramids, 2000, Kees Veenenbos

Pyramids

- The shaded face of each pyramid is called the base of the pyramid.

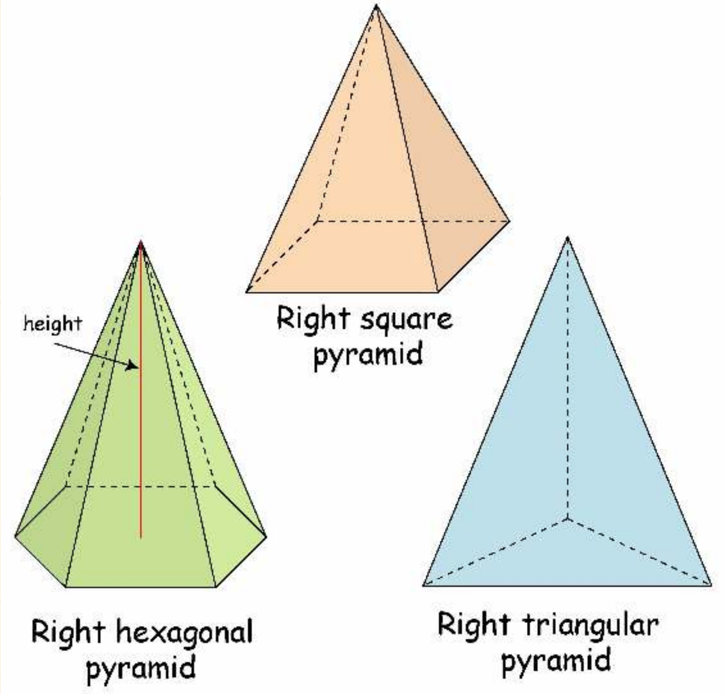
- The faces that are not bases are called lateral faces. Lateral faces meet to form lateral edges.



- The common vertex of the lateral faces is the apex of the pyramid.

Pyramids

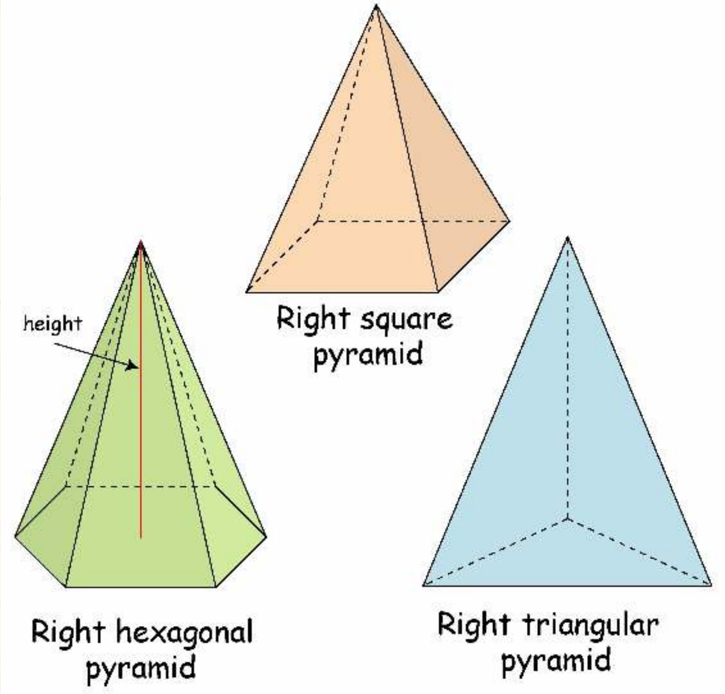
- Pyramids are classified by their bases.



- The pyramids of Egypt are right square pyramids because they have square bases and the apex is directly above the middle of the square.

Pyramids

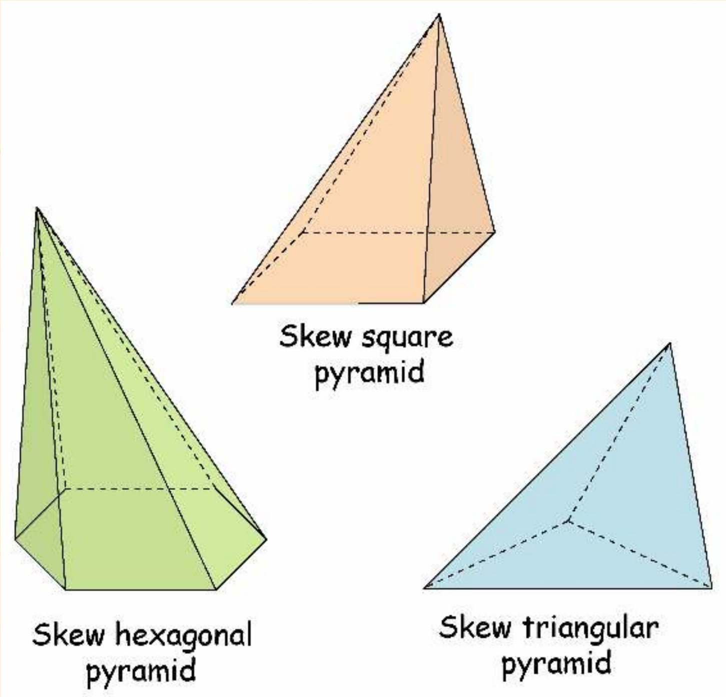
- The distance of a perpendicular line from the apex to the plane of the base is called the height or altitude of the pyramid.



Pyramids

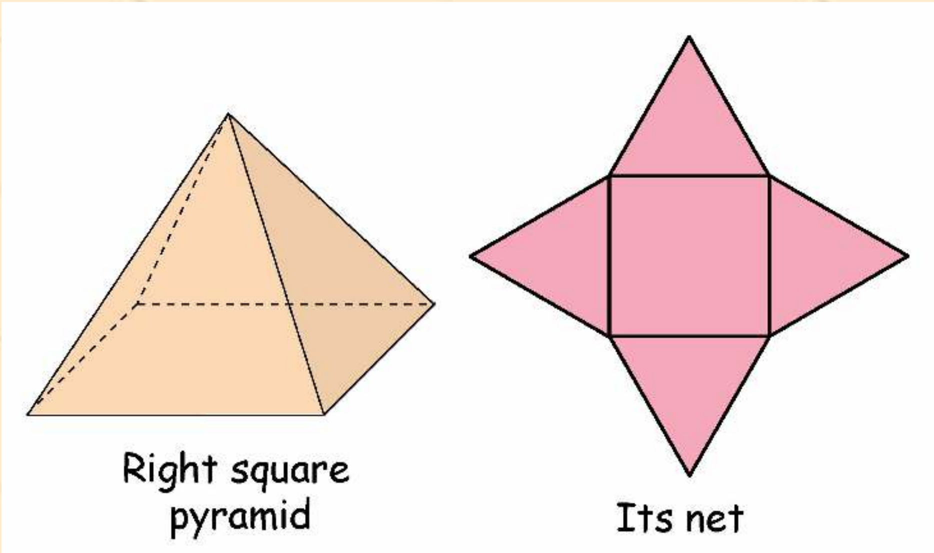
- The apex isn't over the middle of the base in a skew pyramid.

- The height is still the distance of a perpendicular line from the apex to the plane of the base.



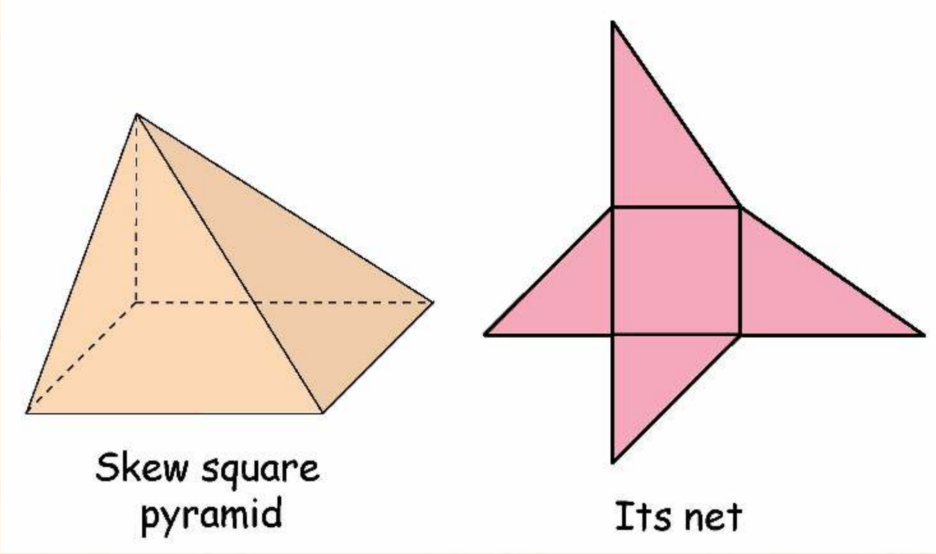
Pyramids

- Below is a diagram called a net for a right square pyramid: a two-dimensional drawing of the faces of the pyramid that can be cut out and taped together to build it.



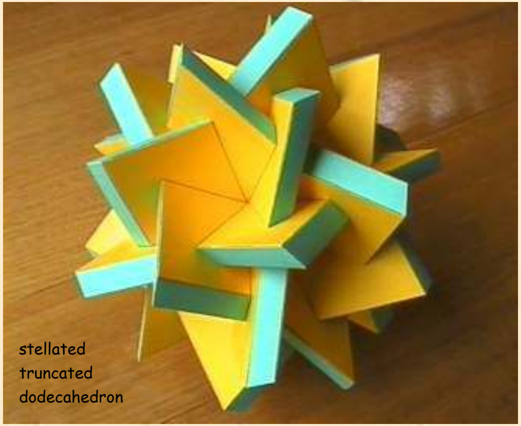
Pyramids

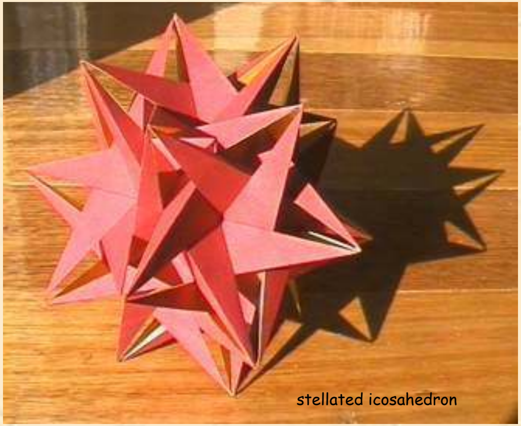
- A net has the advantage that all the polygonal faces are presented without distortion inherent in a perspective drawing.



Pyramids

- Nets are one way how people construct physical realizations of polyhedrons.

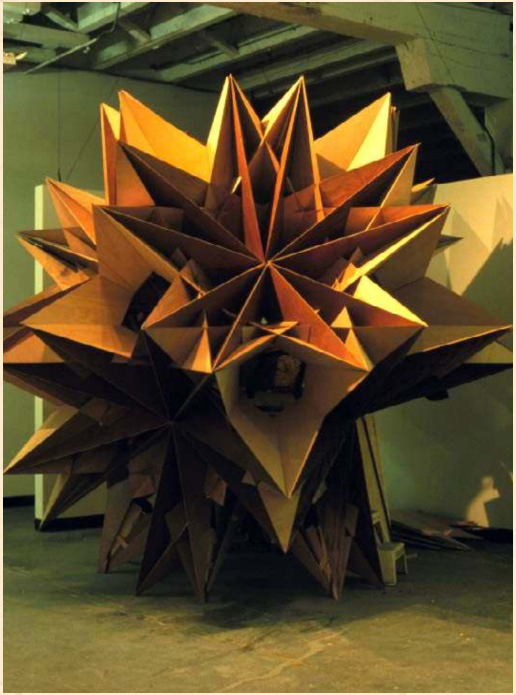


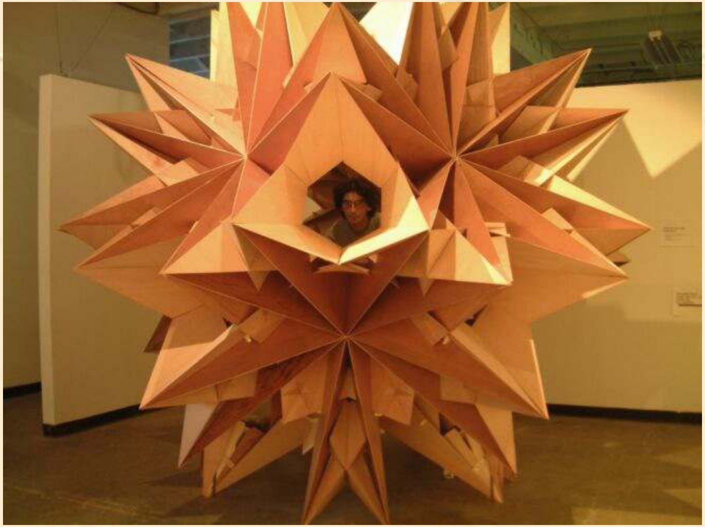


The stellated truncated dodecahedron was probably never been seen or made before, until Robert Webb created it with his net program Stella. All the faces have 5-fold rotational symmetry, being parallel to the faces of the dodecahedron.

Pyramids

- Sometimes polyhedron nets turn into 12 foot high sculptures.

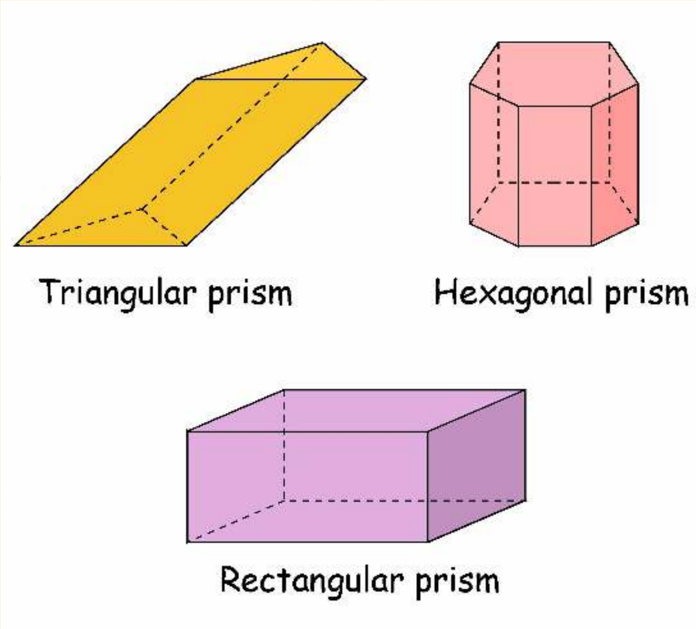




Tom Lechner's "Small Inverted Retrosnub Icosicosidodecahedron".

Prisms

- Prisms are another family of polyhedra.



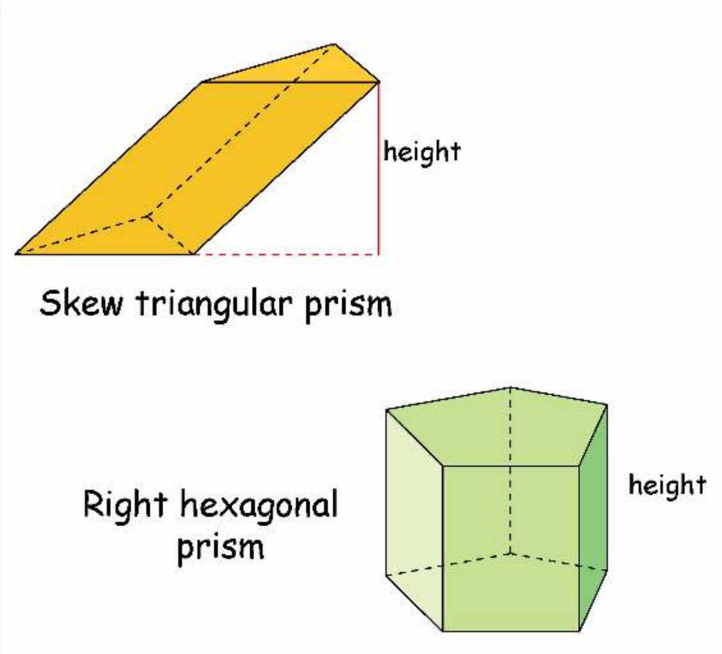
- A prism has two bases that are congruent and parallel polygons, whose other lateral faces are formed by connecting the vertices of the bases.

Prisms

- A prism whose lateral faces are rectangles, implying the lateral edges are perpendicular to the bases, is called a right prism.

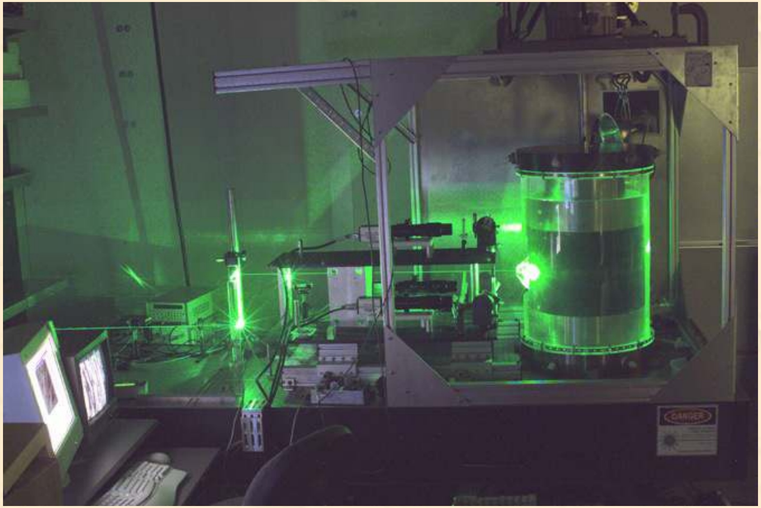
- A prism that isn't a right prism is called a skew prism.

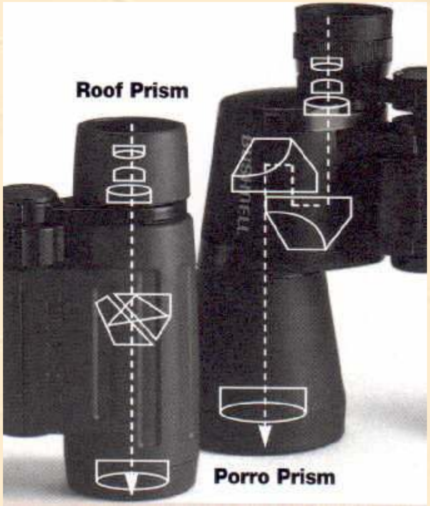
- The height of a prism is the shortest distance between their base planes.



Prisms

- Prisms are particularly relevant in optical systems.





Prisms

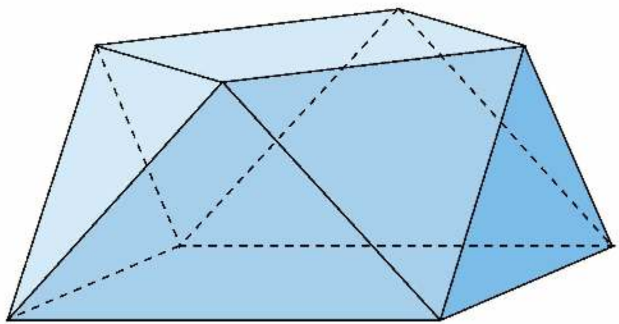
- Astronomy in particular is quite fond of using the refractive properties of prism-like devices to examine the light from planets, stars, and galaxies.

Antiprisms

- Another class of polyhedra are antiprisms.

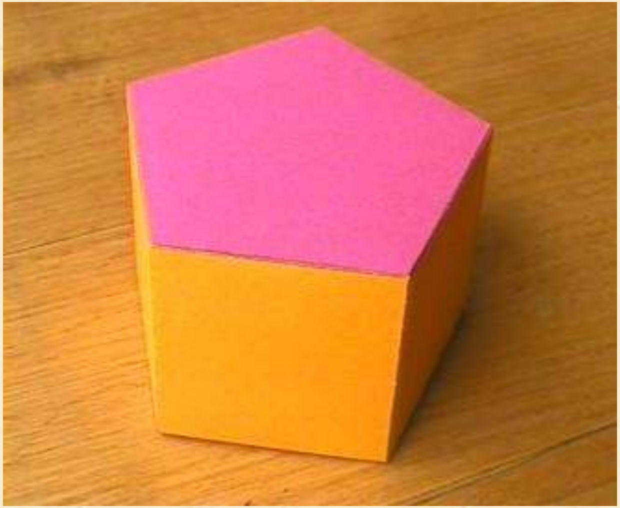
- Antiprisms also have congruent and parallel polygons for bases, but one of the bases is rotated so that its vertices lie over the midpoints of the edges of the other base.

- The vertices of the two bases are then joined by a band of triangles.

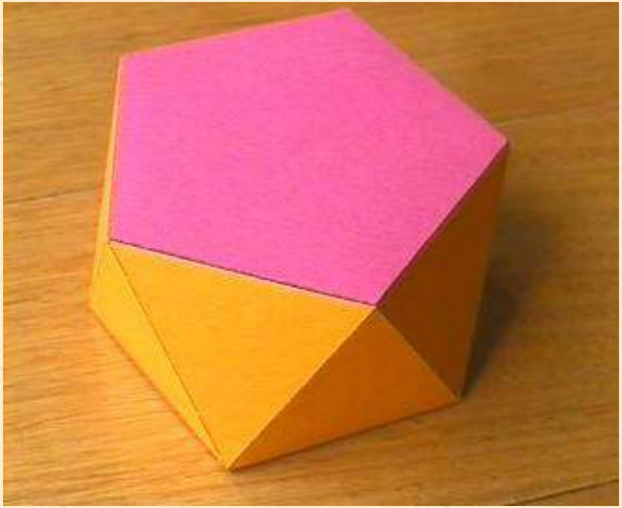


Rectangular antiprism

Antiprisms



Pentagonal Prism



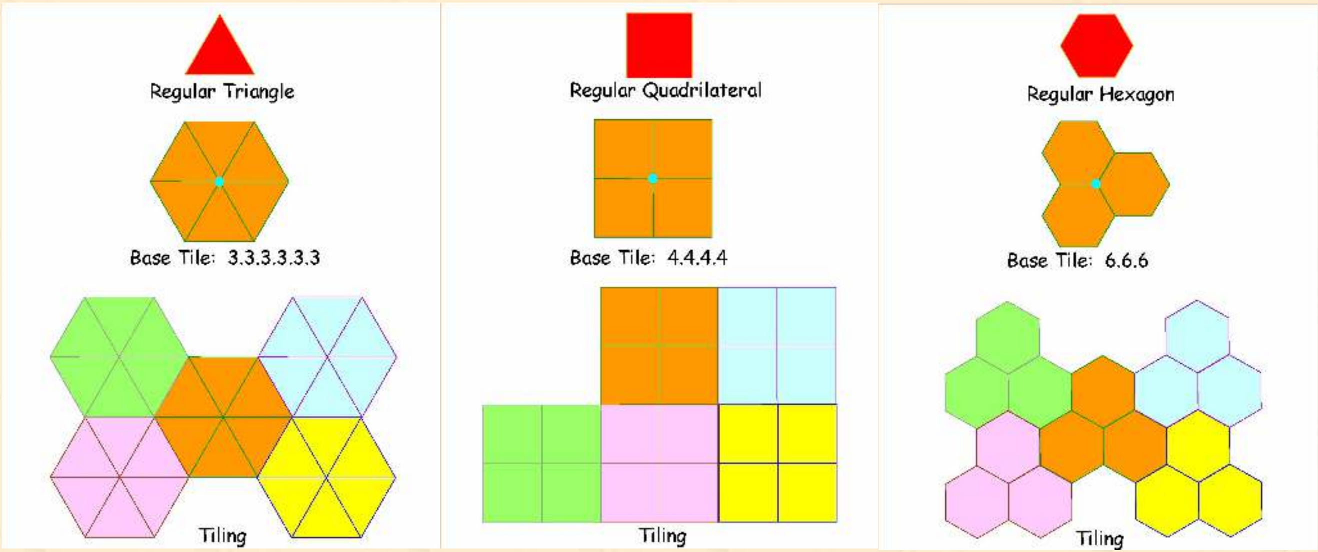
Pentagonal Antiprism

Antiprisms

- A ray traced pentagrammic antiprism.

Platonic solids

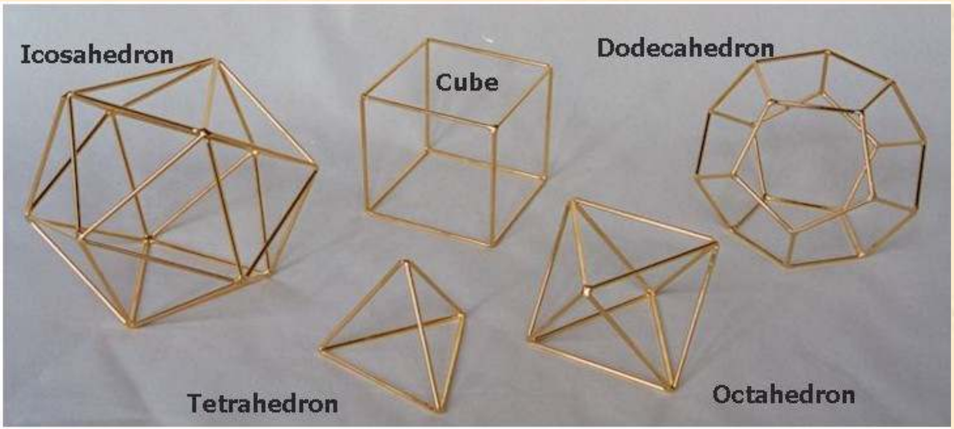
- We've found that only three regular polygons will tile the plane.



- We now wish to find the three-dimensional analogues of the regular tilings.

Platonic solids

- The five regular polyhedra that fill a 3-space are called the Platonic solids.



Platonic solids in 24 K gold plated bronze

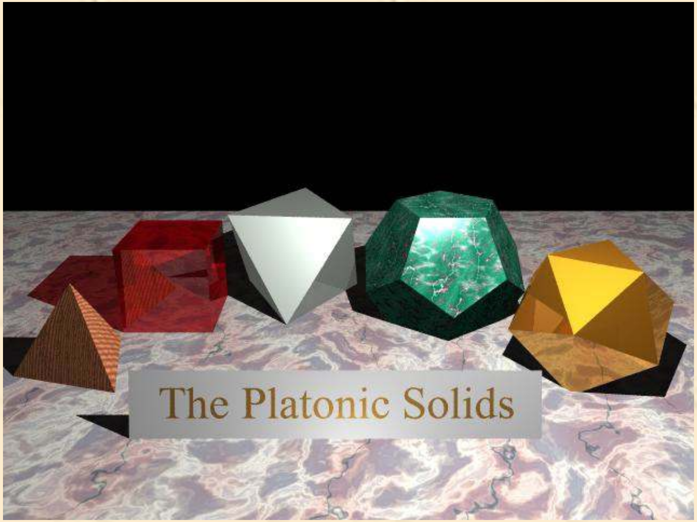
Platonic solids

- Plato reasoned that because all objects are three-dimensional, their smallest parts, atoms, must be in the solid shape of regular polyhedrons, which are explainable by mathematics.

Center region of Raphel's School of Athens

Platonic solids

- In Plato's view, all things are composed of five different atoms: earth, air, fire, water, and quintessence (cosmos).

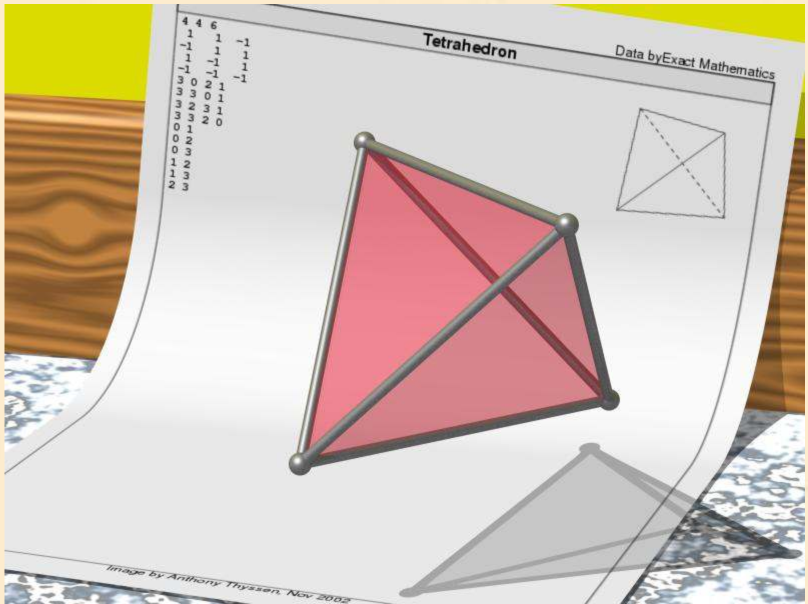


- Plato assigned the shape of each of the five regular polyhedrons that tesselate space to each of the five atoms.

Platonic solids

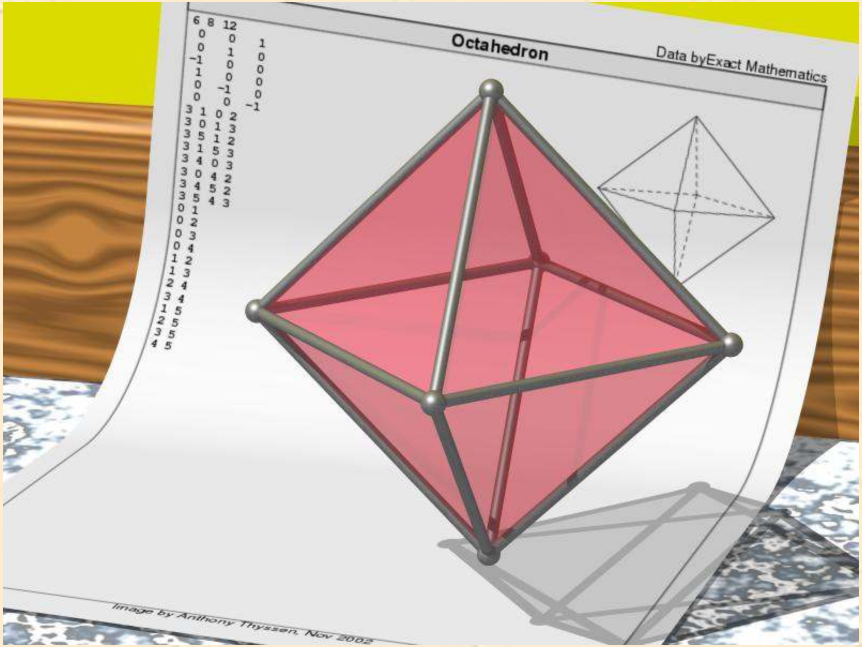
- Plato argued that fire atoms are regular tetrahedra because fire is the lightest atom and the tetrahedron has the least number of faces.

- In addition, the tetrahedron has the sharpest points and, thus, must be responsible for the sharp sting of fire.



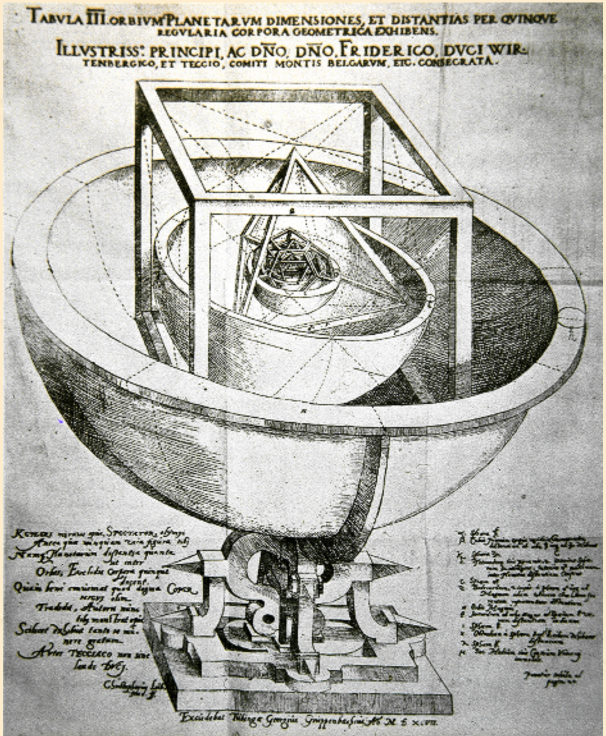
Platonic solids

- It followed that since air is the second lightest, it must be made of octahedra because the octahedron has the second least number of faces.



Platonic solids

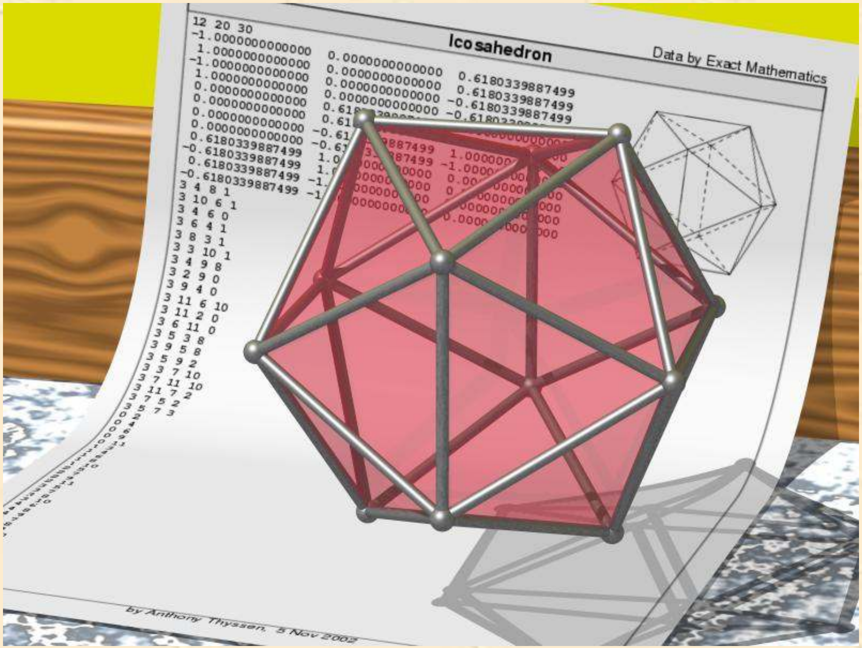
- Plato further reasoned that because fire, air, and water react most often with one another, they must be composed of atoms that are similar in shape.



Mysterium cosmographicum, Kepler, 1596,

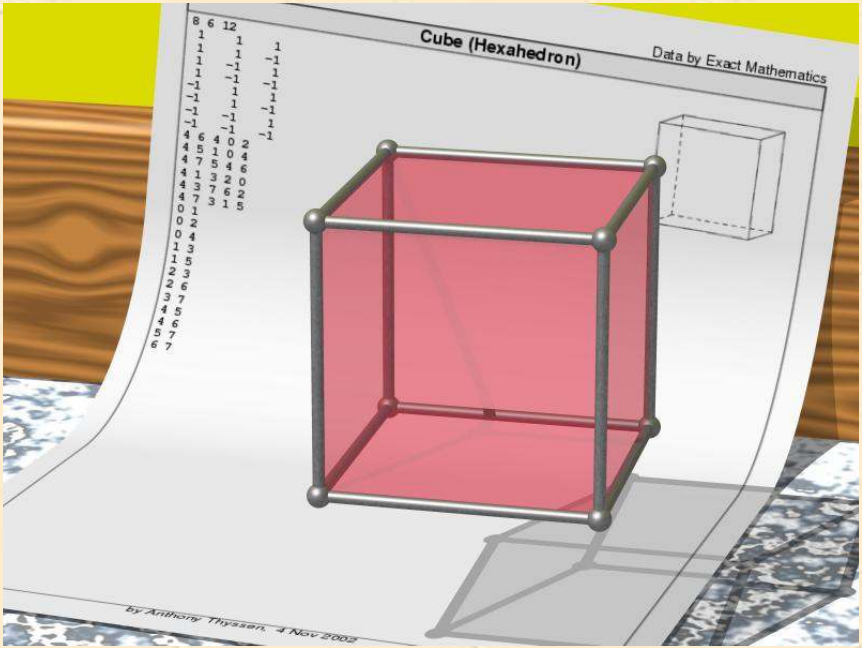
Platonic solids

- Since the icosahedron (20 faces) is the last remaining regular polyhedron composed only of triangles, the water atom is in the shape of the icosahedron.



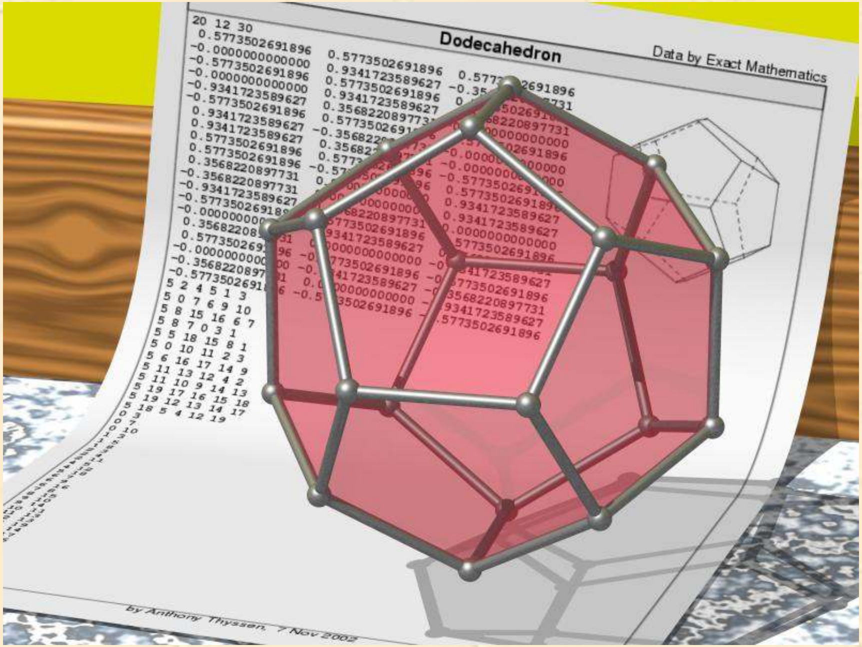
Platonic solids

- Plato then reasoned that the earth atoms are in the shape of cubes, or hexahedrons, because the cube is very stable, like Earth.



Platonic solids

- The fifth and remaining regular polyhedron, the dodecahedron (12 faces), is so unlike the others, having pentagonal symmetries, that Plato argued it must be the shape of the atoms of the cosmos.



Playtime

- During today's construction, you'll create models of the Platonic solids by using Zome tools and nets.