Let no one destitute of geometry enter my doors.

Plato

School of the Art Institute of Chicago

Geometry of Art and Nature

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flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

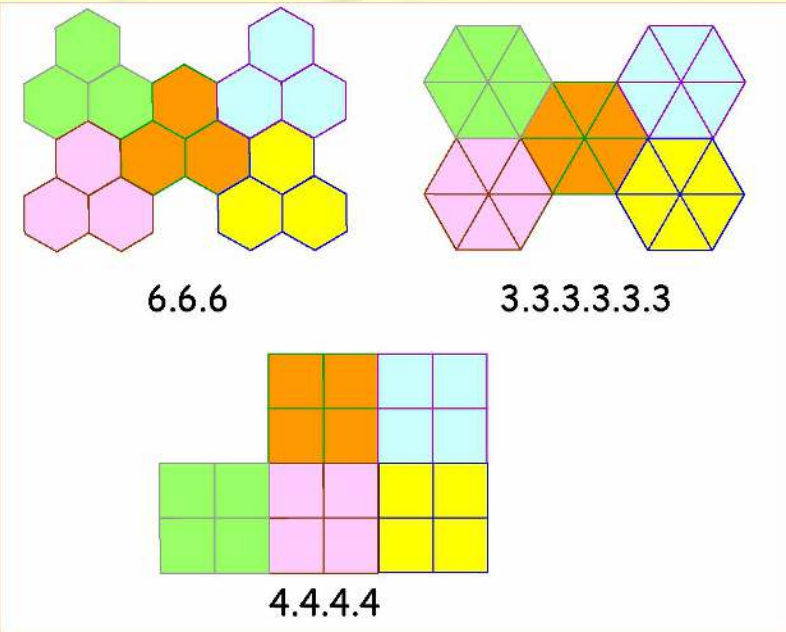
- www.scienceu.com/geometry/facts/solids/
- gratrix.net/polyhedra/uniform/poly_unif_main.html
- home.connexus.net.au/~robandfi/Archimedean.html

Class #9

- Archimedean solids
- Polyhedra in the arts

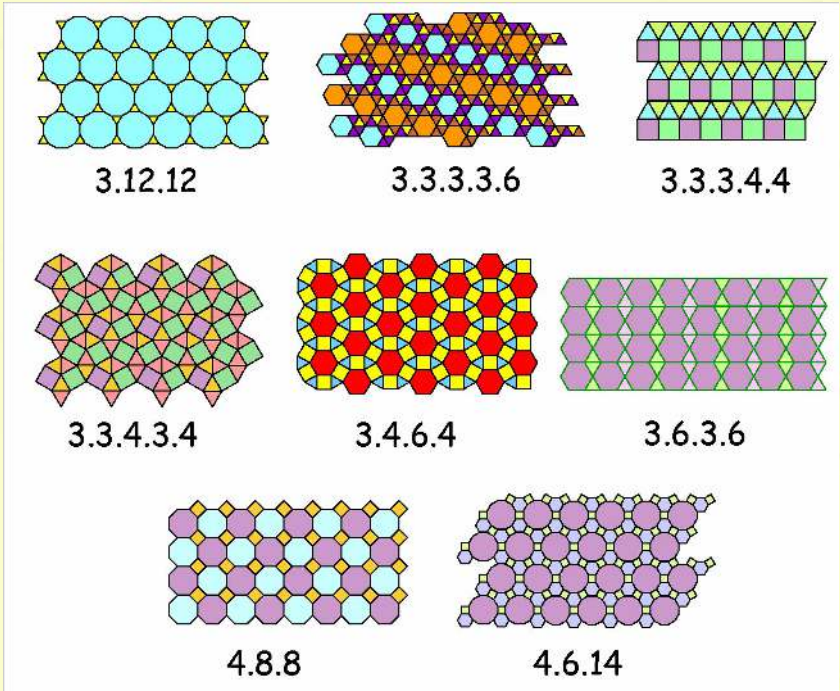
Archimedean solids

- After we found the three regular tilings of the plane by squares, equilateral triangles, and hexagons …



Archimedean solids

- … we went on to find the eight semiregular tilings.



Archimedean solids

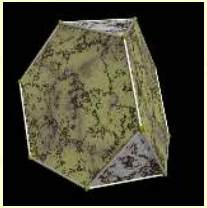
- The analogs of the regular tilings for polyhedra are the five Platonic solids; the tetrahedron, cube, octahedron, dodecahedron, and icosahedron.

- Now we want to build the three-dimensional analogs of the semiregular tilings.

Archimedean solids

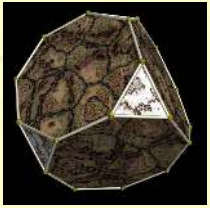
- The 13 semiregular, or Archimedean, polyhedra have faces of regular polygons, though more than one type of polygon may be used, and each vertex has the same polygons occuring in the same order.

Archimedean solids



Truncated
Tetrahedron

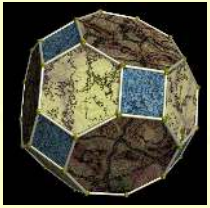
- These solids have lots of symmetries amongst them;
  one has tetrahedral symmetry,
  six have cub-octahedral symmetry, and
  six have icosa-dodecahedral symmtery.



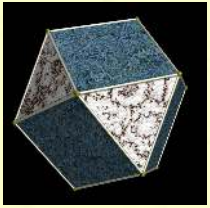
Truncated
Cube

Rombicuboctahedron

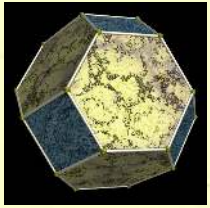
Snub
Cube



Truncated
Cuboctahedron



Cuboctahedron



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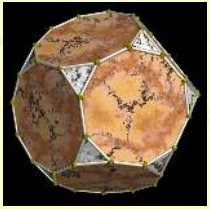
Octahedron



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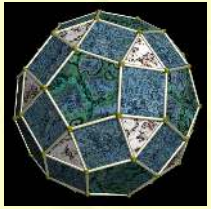
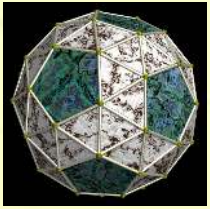
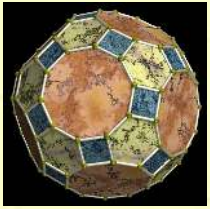
Dodecahedron



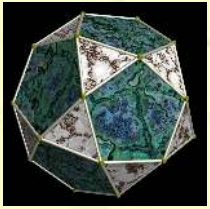
Rhombicosa-
dodecahedron



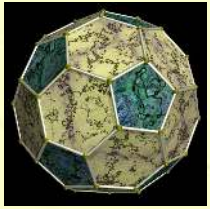
Snub
Dodecahedron



Truncated
Icosadodecahedron

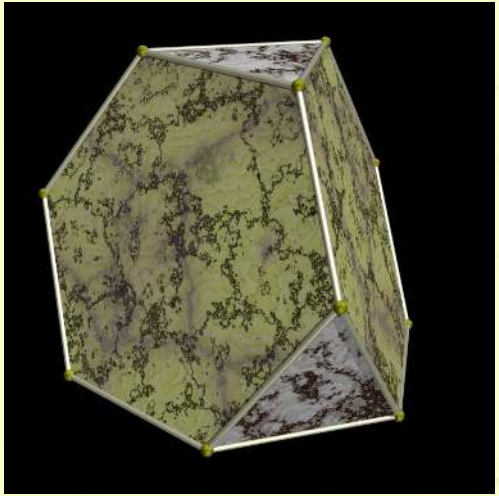


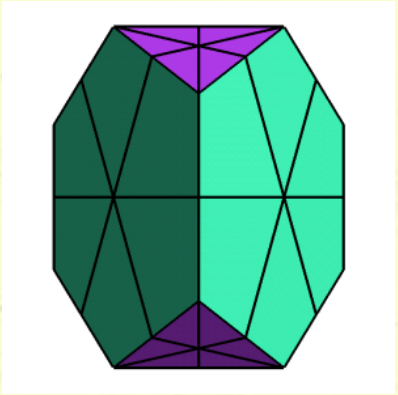
Icosadodecahedron

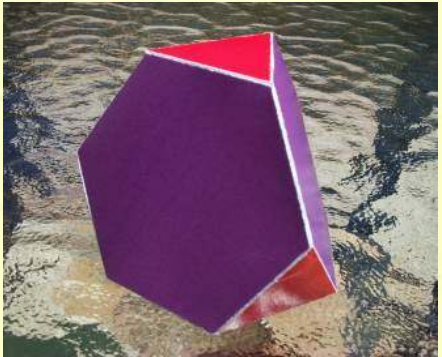


Truncated
Icosahedron

Truncated Tetrahedron

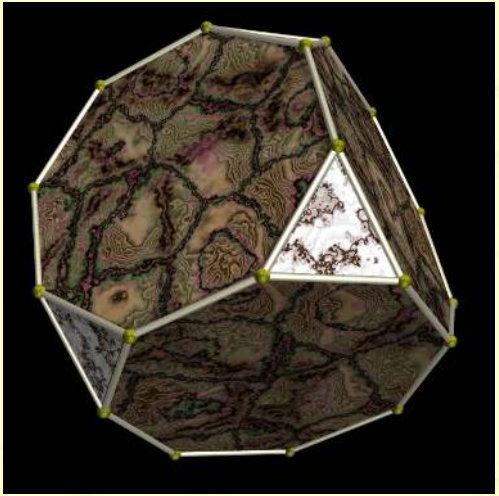


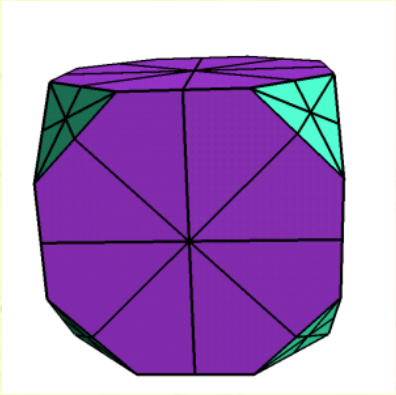


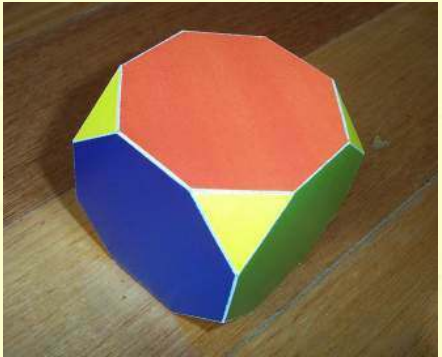


Vertex: 3.6.6

Truncated Cube







Vertex: 3.8.8

Polyhedra in art

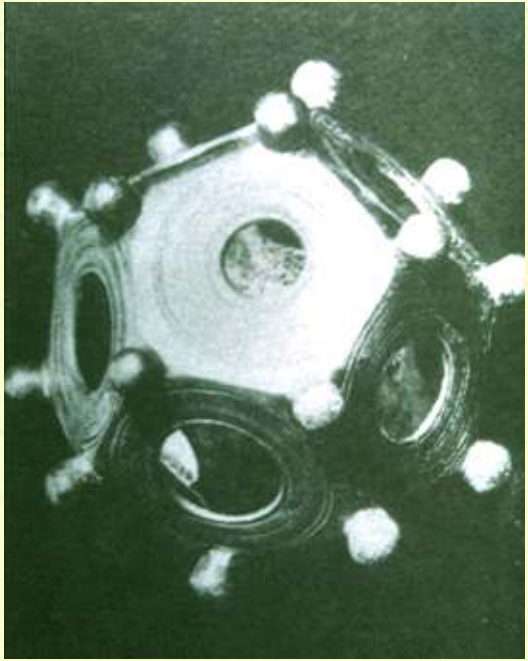
- Polyhedra are nothing new.

Secrets of the Great Pyramid
1971, Cover Art, Peter Tompkins

- The Egyptians, of course, knew of the tetrahedron. But also of the octahedron, the cube, and there are icosahedral dice from the Ptolomaic dynasty in the British Museum, London.

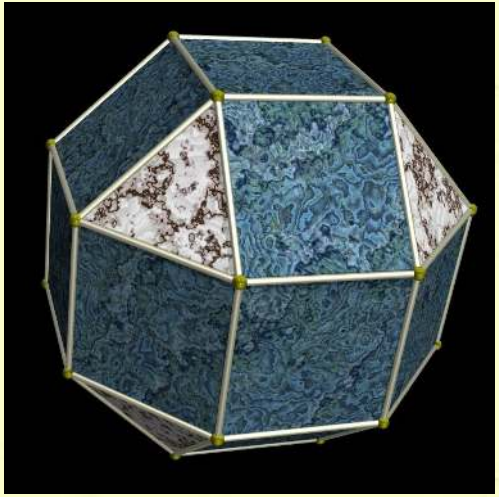
Polyhedra in art

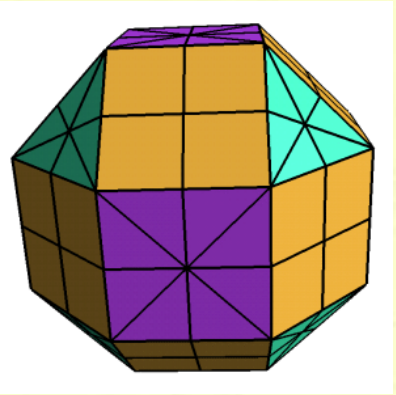
- There are Neolithic solids found in Scotland, and excavations near Padova have unearthed an Etruscan dodecahedron, 500 BC, probably used as a toy.

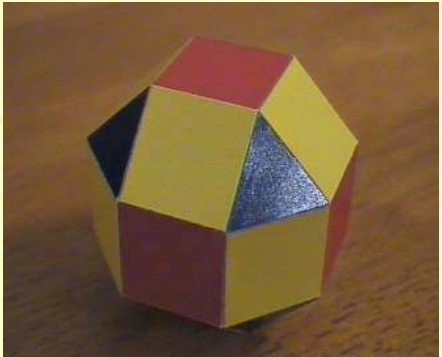


The Visual Mind: Art and Mathematics
1993, Michele Emmer

Rhombicuboctahedron



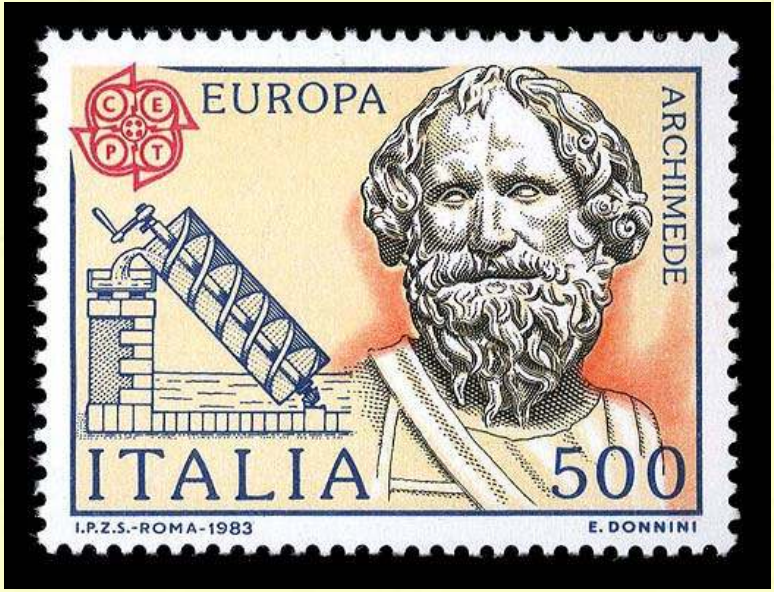




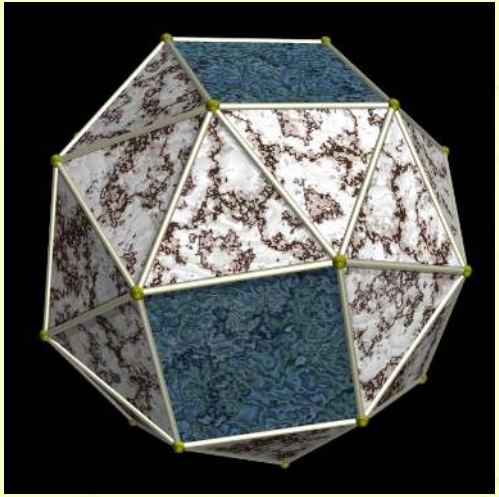
Vertex: 3.4.4.4

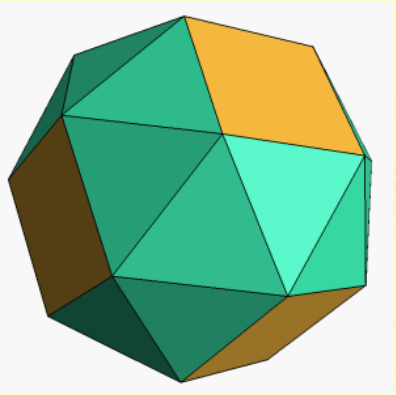
Archimedean solids

- While Archimedes apparently knew of all thirteen of the solids that bear his name, his book on the subject was lost many centuries ago.



Snub Cube





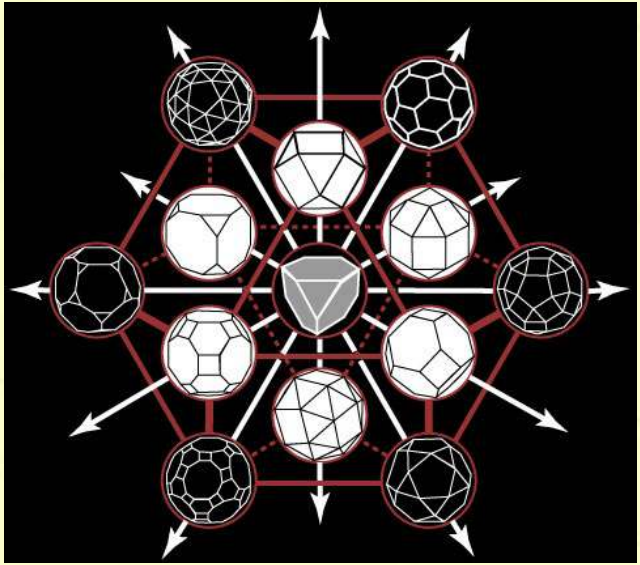
Vertex: 3.3.3.4

Renaissance polyhedra

- This painting shows Fra Luca Pacioli and his student, Guidobaldo, Duke of Urbino. In the upper left is a rhombicuboctahedron and on the table is a dodecahedron on top of a copy of Euclid's Elements.

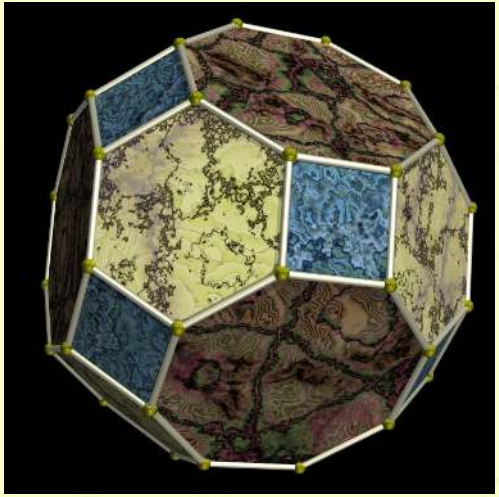
Luca Pacioli
1499, Jacopo de Barberi

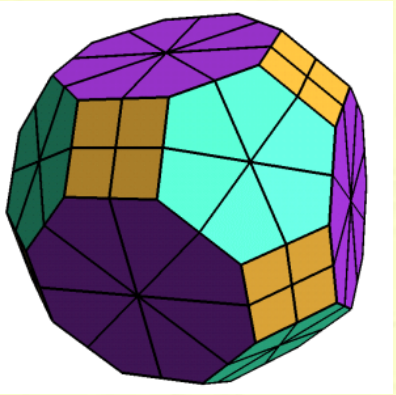
Archimedean solids

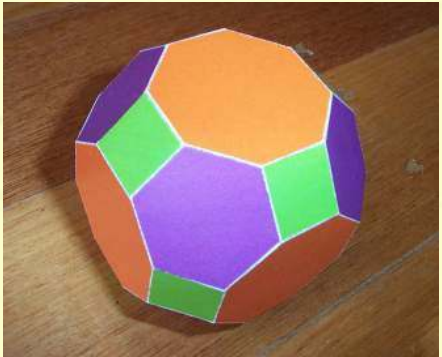


… The 12 white vectors of Fuller's Dymaxion (Vector Equilibrium) pass from the center of the truncted tetrahedron each one through one of its 12 vertices and on through each of the Archimedians residing at the 12 vertices of the cuboctahedron. The truncated tetrahedron is the only semi-regular solid figure with 12 independent axis (The Twelve Degrees of Freedom) passing through its vertices from its center. As to the question of whether Existence is or is not a matter of randomness, PORCELAINia is more inclined to answer, "is not" …

Truncated Cuboctahedron



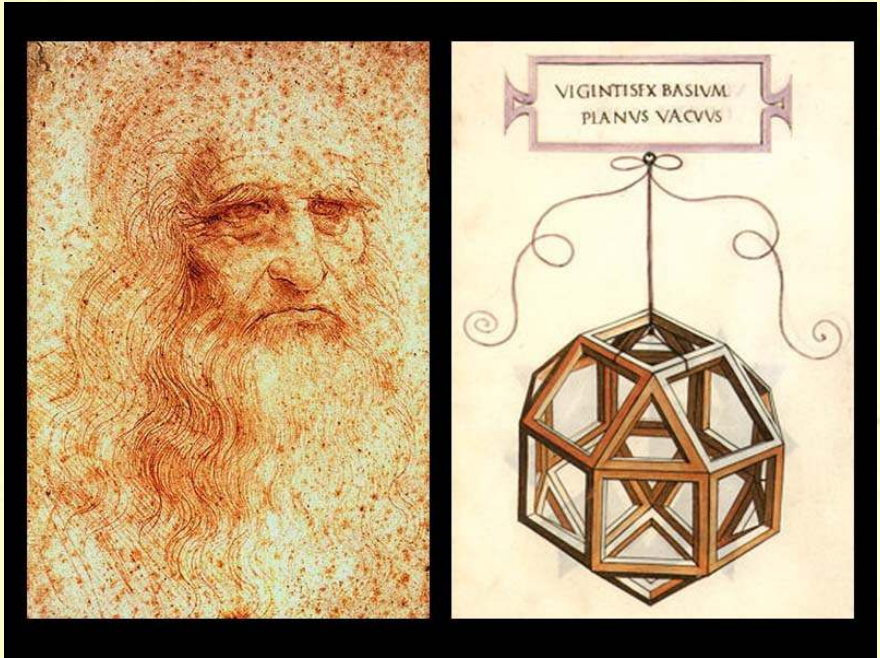




Vertex: 4.6.8

Renaissance polyhedra

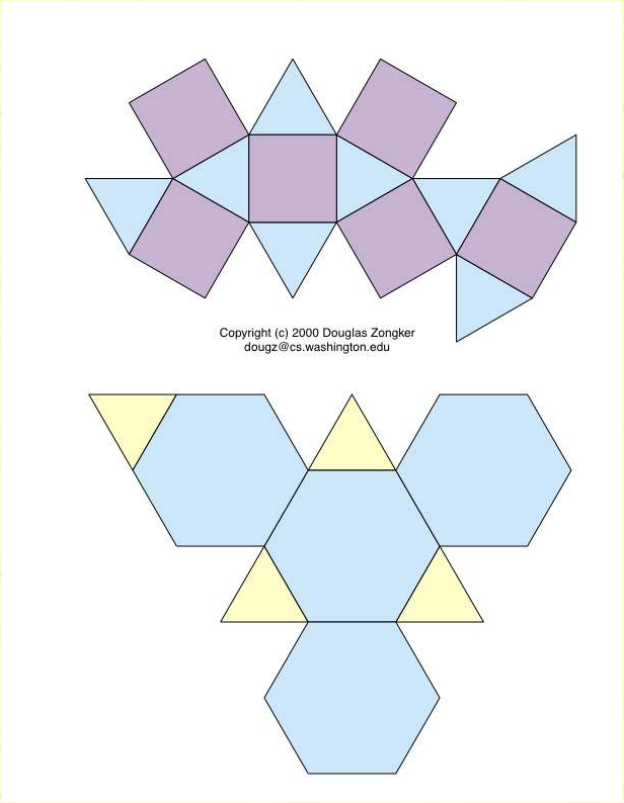
- Luca Pacioli wrote a book called Da Divina Proportione (1509) which contained a section on the Platonic Solids and other solids. This text has 60 plates of solids illustrated by his student Leonardo da Vinci.

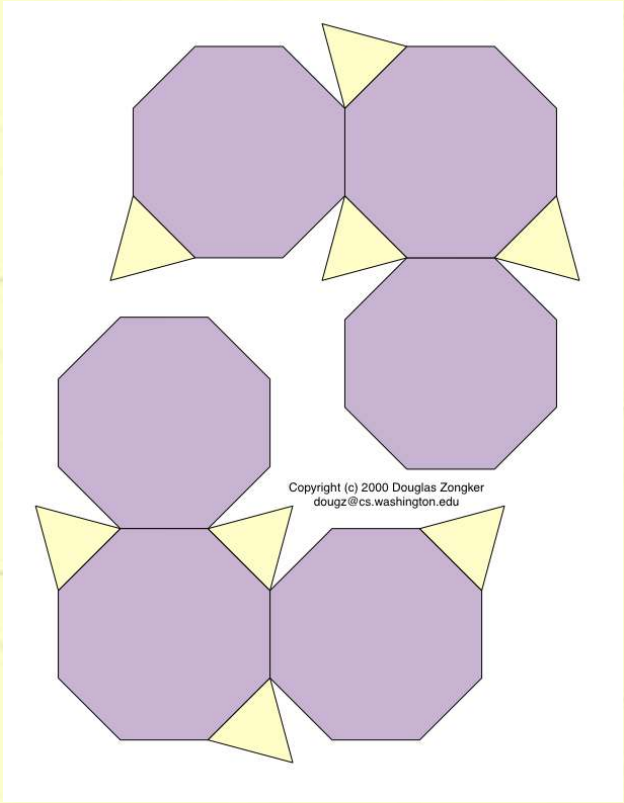


Renaissance polyhedra

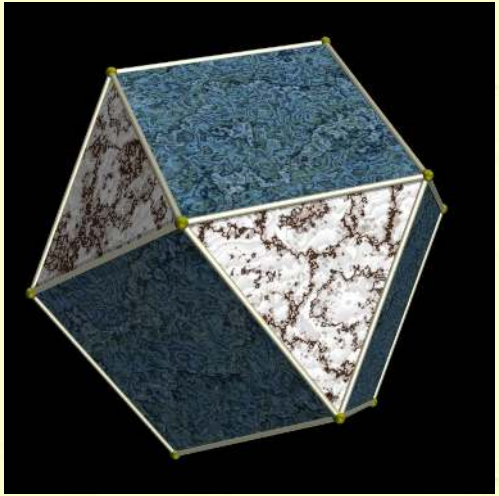
Mosaic from San Marco Cathedral
Venice

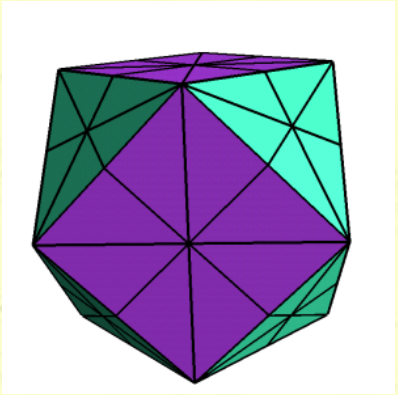
Archimedean solids

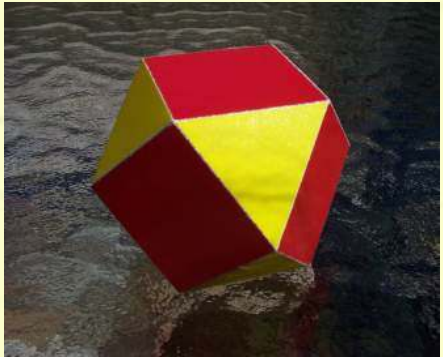




Cuboctahedron







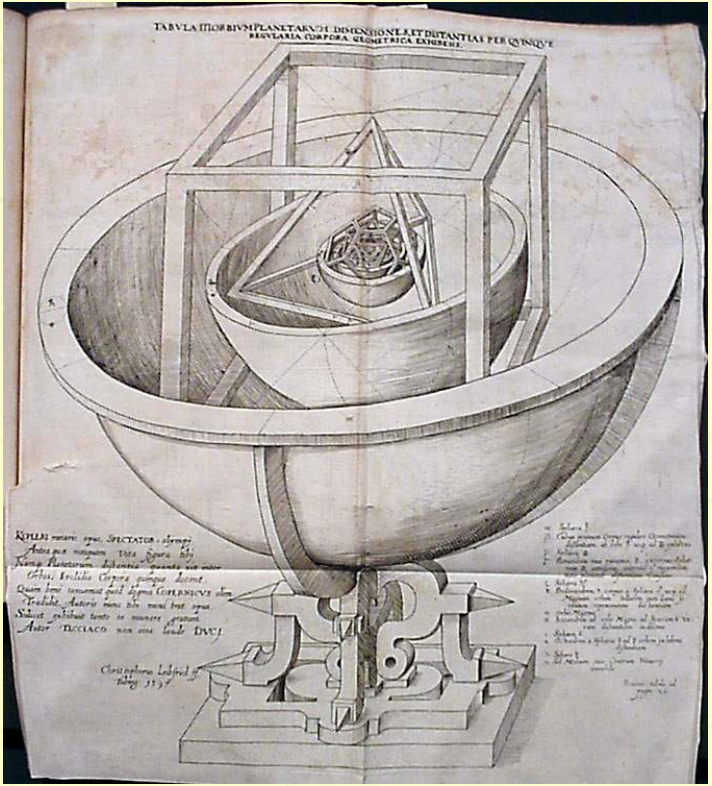
Vertex: 3.4.3.4

Kepler

- While several of the semiregular polyhedra appear in text and images of the Renaissance, the complete list was rediscovered by Kepler, who also defined the classes of prisms and antiprisms that we looked at last time.

Kepler

- In 1596 Kepler published "The Cosmic Mystery" in which he envisioned the universe as consisting of nested Platonic Solids whose inscribed spheres determine the orbits of the planets, all enclosed in a sphere representing the outer heaven.



Kepler

- Later, Kepler found the true motions of the planets. These are enshrined in astronomy as Kepler's Laws.

Kepler

1. The orbits of the planets are ellipses, with the Sun at one focus.

2. As a planet moves around its orbit, it sweeps out equal area in equal time.

3. A planet's sidereal period squared is proportional to its semimajor axis cubed.

$$P^2 = a^3$$

P = time to complete orbit, in years

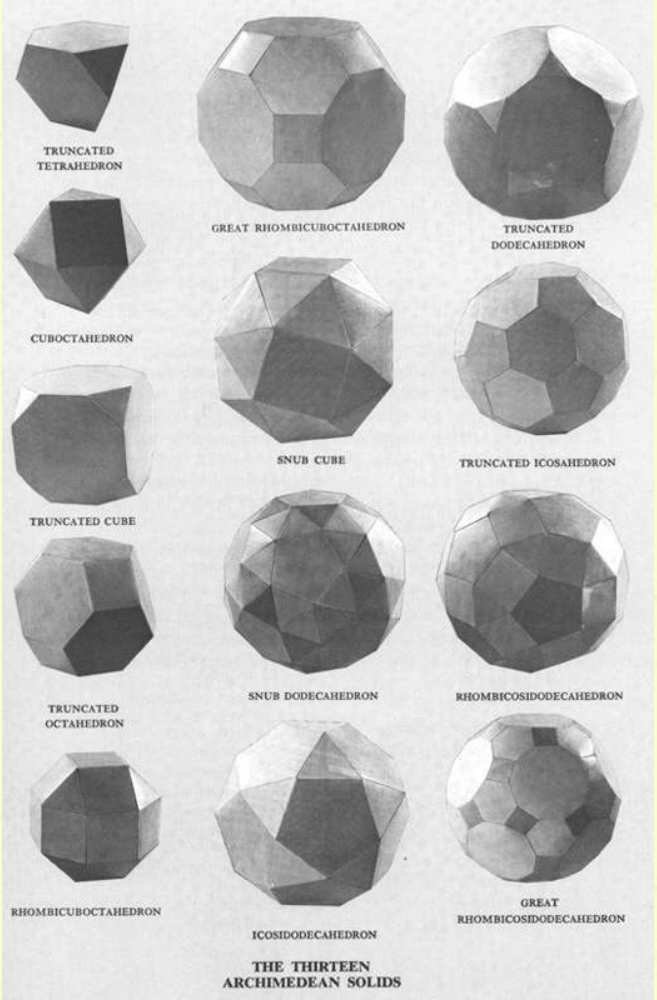
a = semimajor axis, in Astronomical Units

1 AU = average Earth-Sun distance

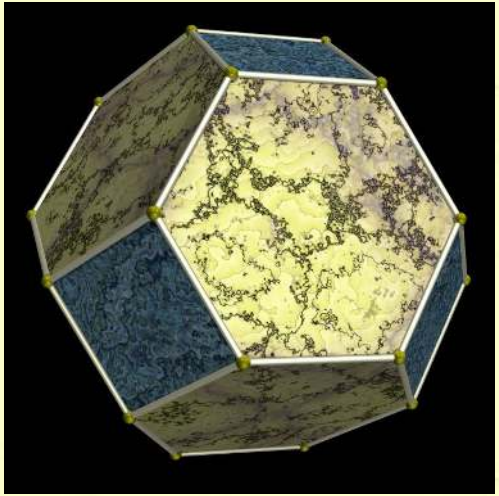
- Many of the names originate from Kepler 's terminology and its translations from his Latin.

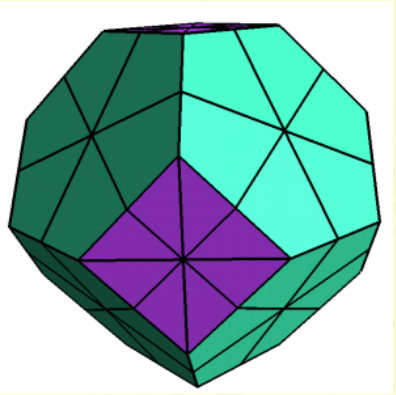
- The term "truncated" refers to the process of cutting off corners.

- The term "snub" refers to a process of replacing each edge with a pair of triangles.



Truncated Octahedron





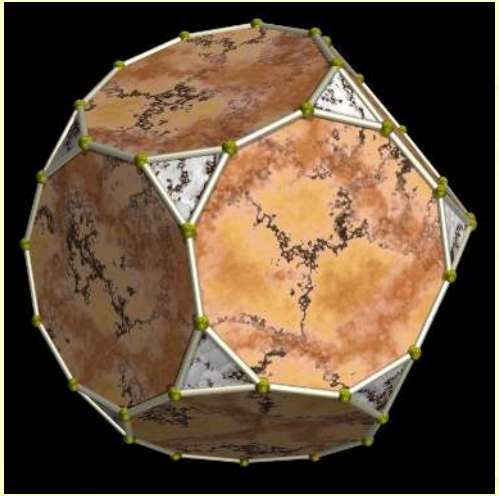
Vertex: 4.6.6

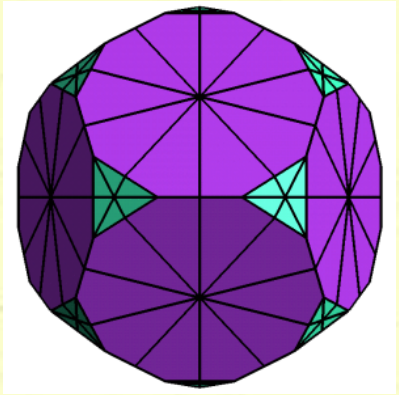
Renaissance polyhedra

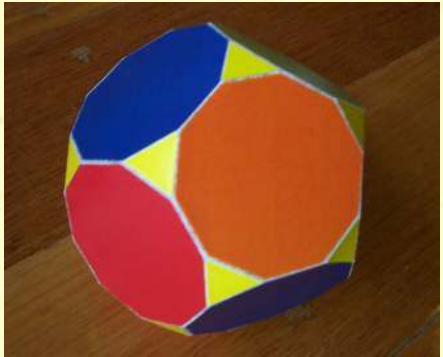
- Albrech Durer had a keen interest in geometry. This famous engraving shows a cube with opposite corners cut off, as well as a sphere, a magic square, and compasses.

Melancolia I
1514, Albrech Durer

Truncated Dodecahedron







Vertex: 3.10.10

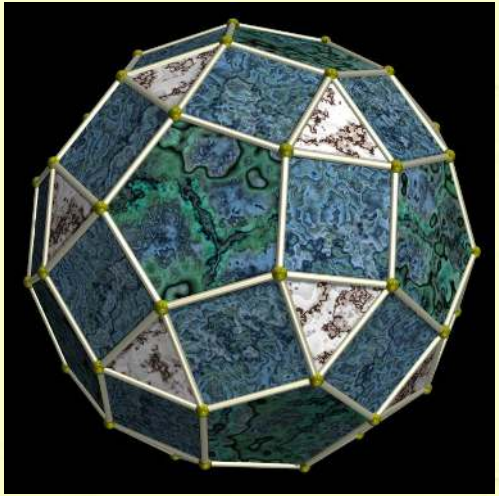
Renaissance polyhedra

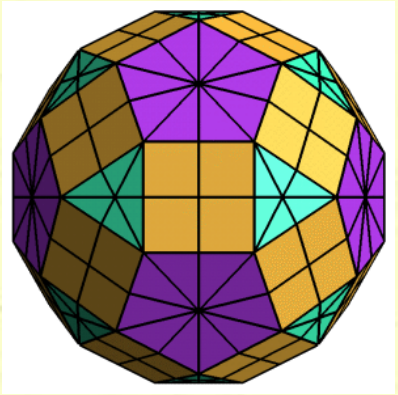
Picture of Johannes Neudorfer and his Son
1561, Nicolaus Neufchatel

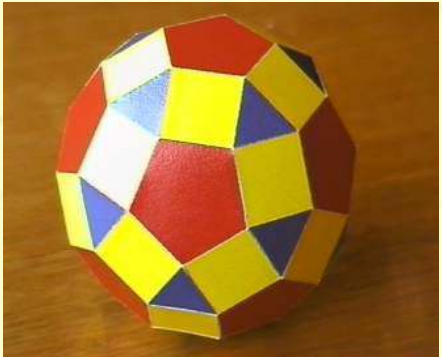
Renaissance polyhedra

Close up of the gold-plated lion inthe front of theGate of Heavenly Purity, Forbidden City, Beijing.
From Qing Dynasty (1736-1796)

Rhombicosadodecahedron



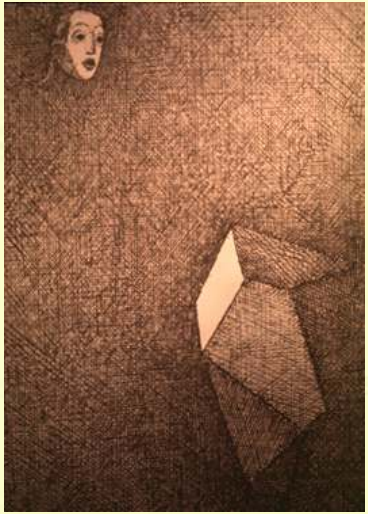




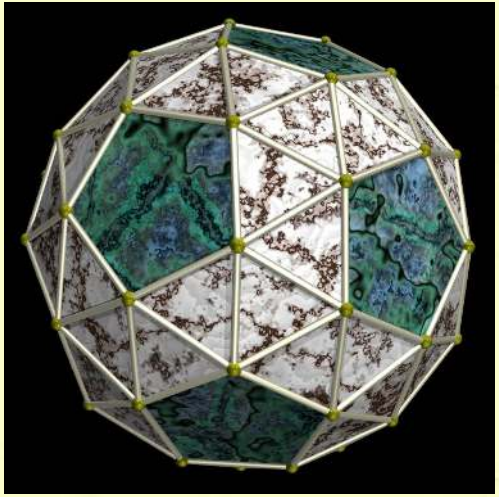
Vertex: 3.4.5.4

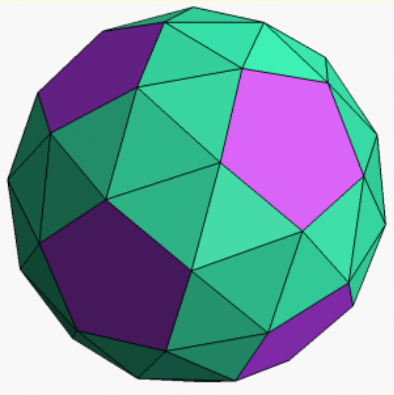
20th century polyhedra

- The Swiss artist Alberto Giacometti (1901-1966) often included polyhedra in his early surrealist works such as these two drawings and a sculpture.



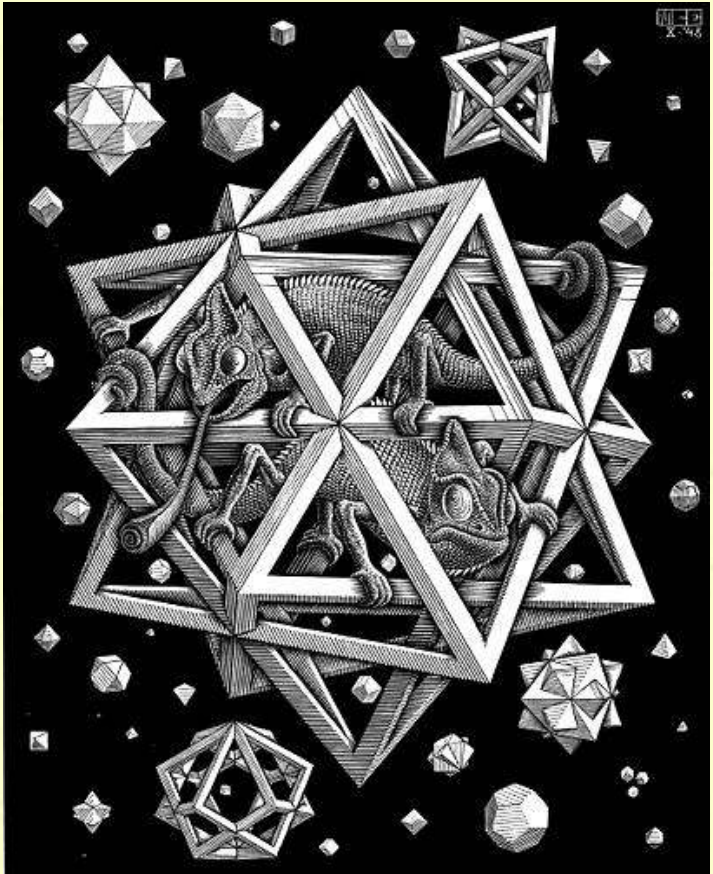
Snub Dodecahedron





Vertex: 3.3.3.3.5

20th century polyhedra



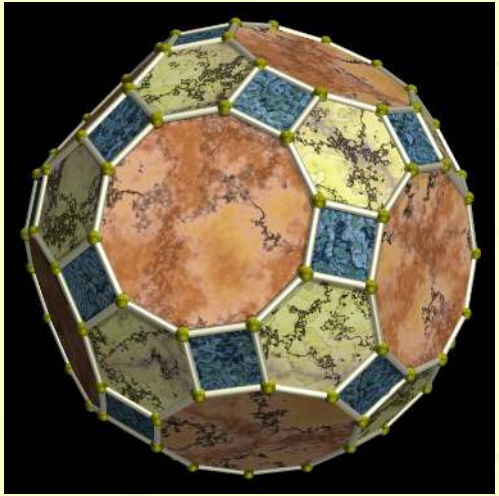
Stars
1948, M.C. Escher

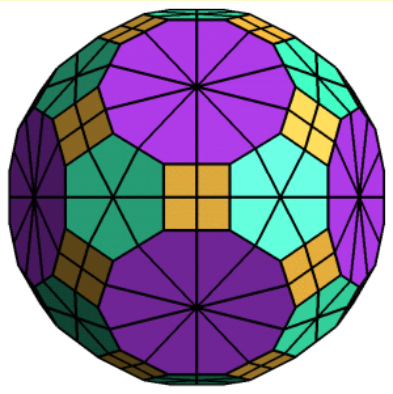
There is a similarity between these polyhedra and Leonardo's illustrations for Pacioli's book.

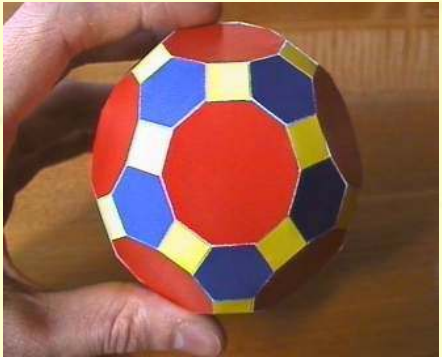
20th century polyhedra

Escher contemplating his
nested set of Platonic Solids.

Truncated Icosadodecahedron





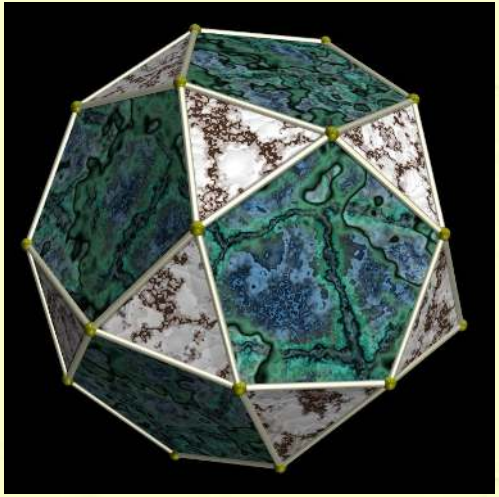


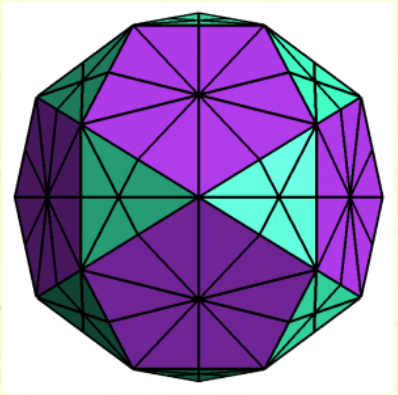
Vertex: 4.6.10

20th century polyhedra

Truncated Close-Packing Octahedra, Rhombidodecahedra, and Cubes.
1980, Harriet Brisson
Plexiglass, aluminum tubes, and nylon cord.

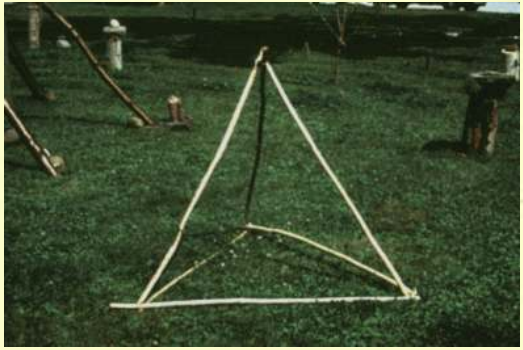
Icosadodecahedron

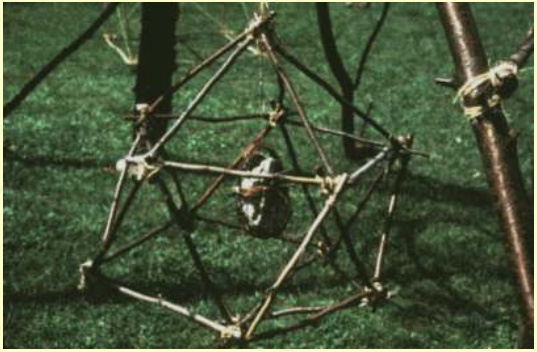




Vertex: 3.5.3.5

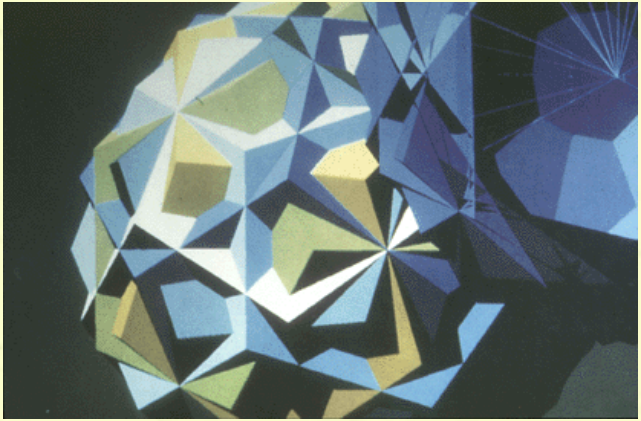
20th century polyhedra

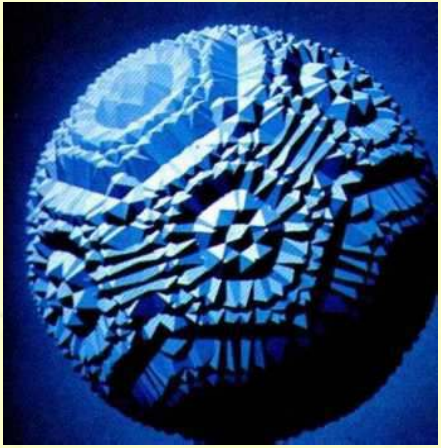


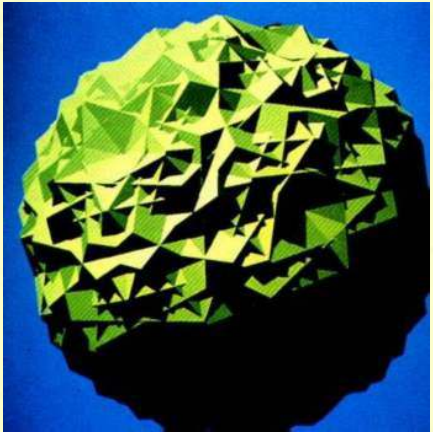


Sorcerer's Circle
1999, Paul Calter

20th century polyhedra

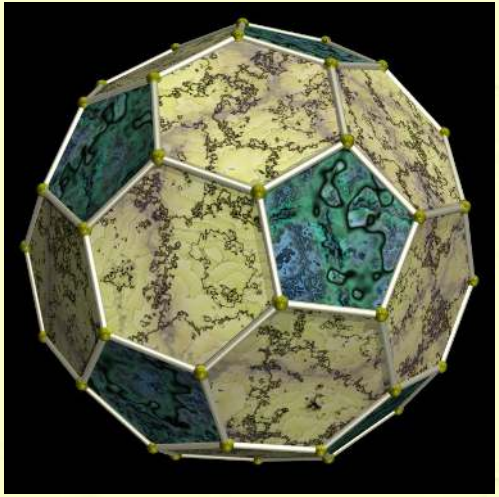


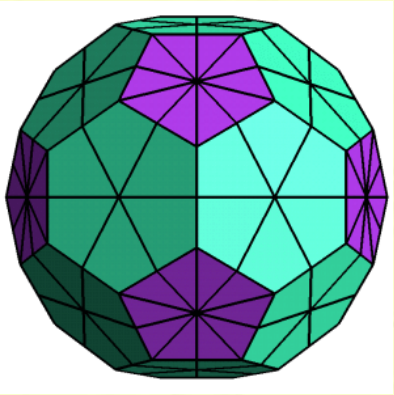


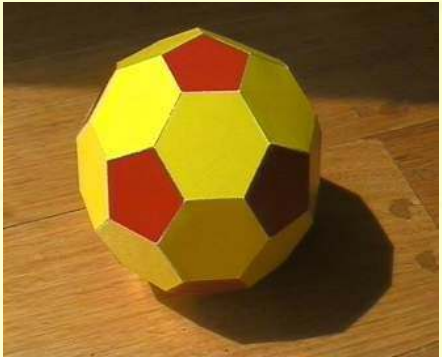


Platonic Forms
1989, Lucio Saffaro

Truncated Icosahedron

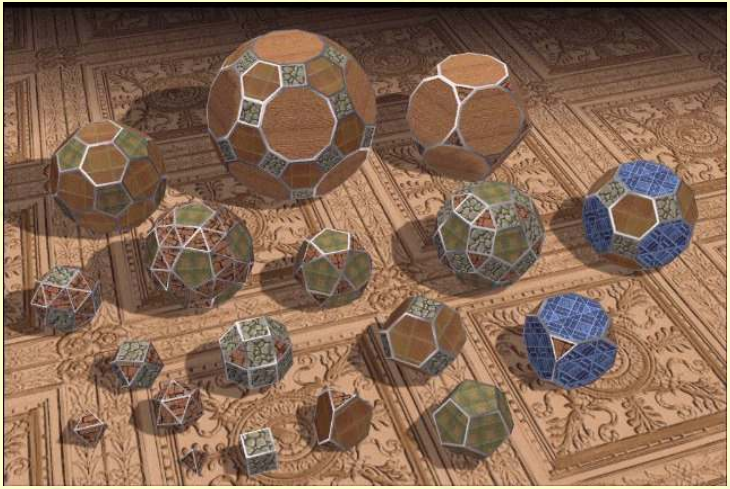


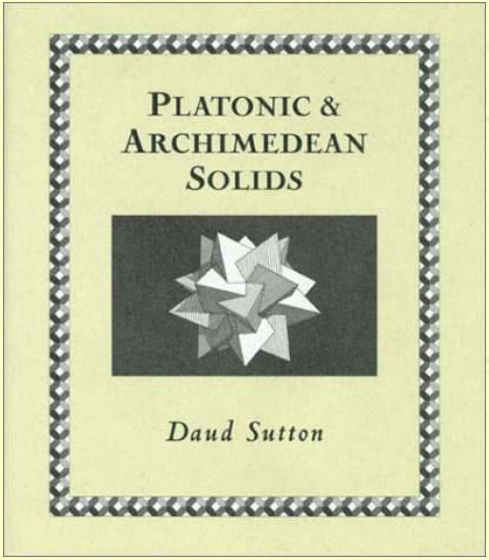




Vertex: 5.6.6

Archimedean solids





Playtime

- During today's in-class construction, you'll create physical models of the Archimedean solids using either the Zome tools or nets.