A ship in a port is safe,
but that is not what ships are built for.

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# Geometry of
# Art and Nature

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flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

# Syllabus

| | | |
|---|---|---|
| 1 | Sept 03 | Basics and Celtic Knots |
| 2 | Sept 10 | Golden Ratio |
| 3 | Sept 17 | Fibonacci and Phyllotaxis |
| 4 | Sept 24 | Regular and Semiregular tilings |
| 5 | Oct 01 | Irregular tilings |
| 6 | Oct 08 | Rosette and Frieze groups |
| 7 | Oct 15 | Wallpaper groups |
| 8 | Oct 22 | Platonic solids |
| 9 | Oct 29 | Archimedian solids |
| 10 | Nov 05 | Non-Euclidean geometries |
| 11 | Nov 12 | Bubbles |
| 12 | Dec 03 | Fractals |

# Sites of the Week

- home.earthlink.net/~marutgers/
- www.exploratorium.edu/ronh/bubbles/
- bubbleart.com/index.html
- www.math.uiuc.edu/~jms/Images/

# Class #11

- More than a toy
- Plateau's observations
- Minimal surfaces
- Antibubbles

# Bubbles

- Most of us were probably blowing bubbles at birth.

# Bubbles

- Later, we probably found bubbles fun to blow and beautiful to contemplate.

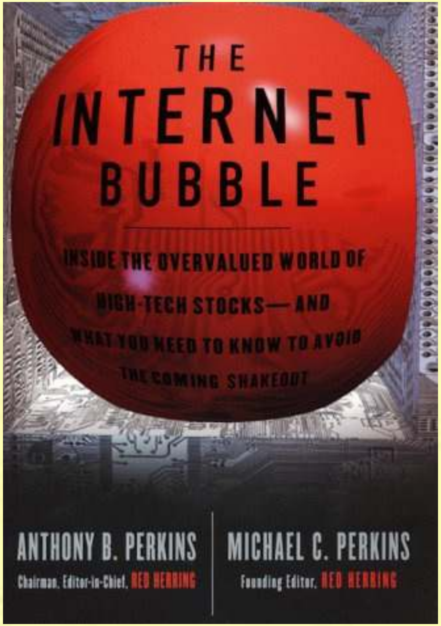
# Bubbles

- Some of us may still enjoy the comforts of a bubbly environment.

- Is that all there is to bubbles??

# Bubbles

- Bubbles are actually big business, worth a lot of money.



# Bubbles

- There's a huge range of systems in industry where a gas of some form gets pumped into a liquid of some form, ranging from sewerage treatment plants to metal smelters.

Metal-bearing minerals are separated from other mineral particles by attaching them to soapy bubbles.

# Bubbles

- The most important aspect of the bubbles is making sure that they're the right size for the job.

- Whether that's in heavy industry or the simple pleasure of a tasty beverage.

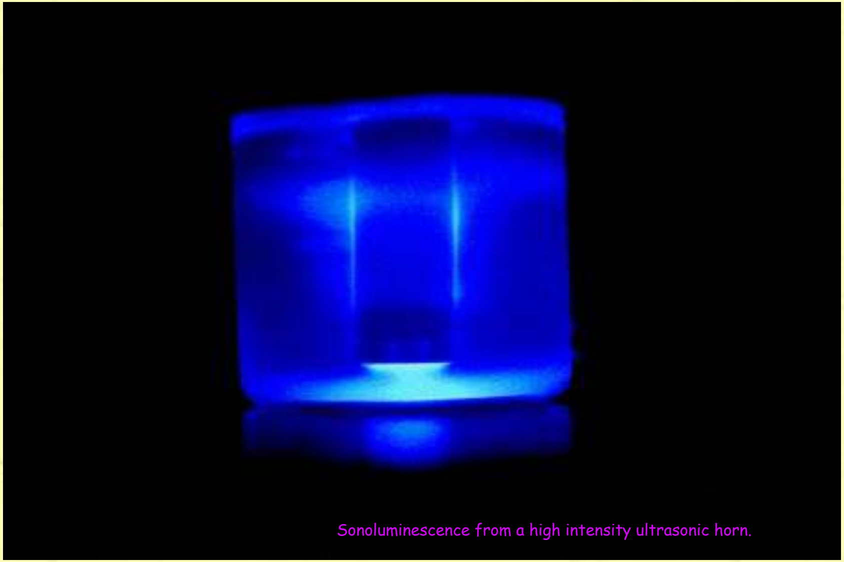
# Bubbles

- Bubbles are also intriguing in chemistry and energy research.

# Bubbles

- When a bubble is driven to collapse on itself, the energy becomes concentrated and can produce a flash of light when the bubble finally pops.

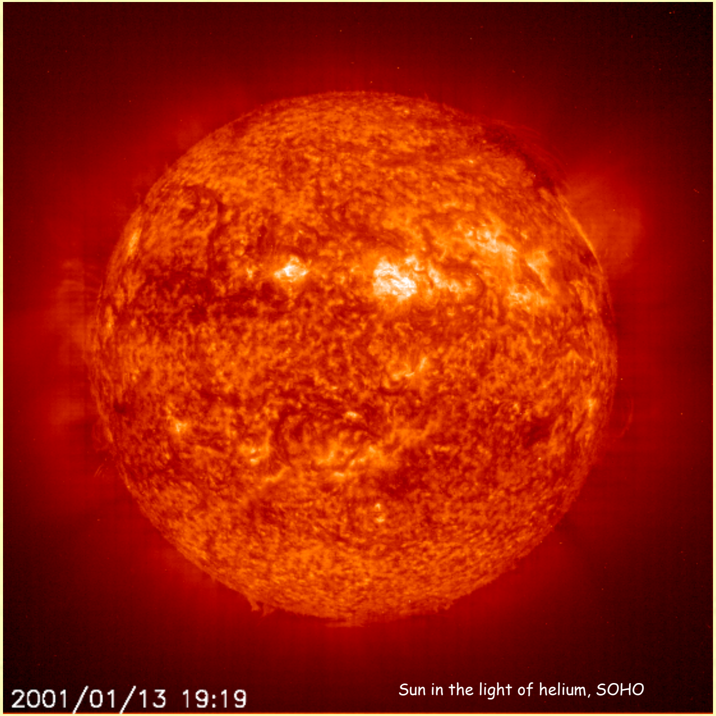
- This phenomena is called sonoluminescence.



Sonoluminescence from a high intensity ultrasonic horn.

# Bubbles

- The collapse of the bubbles produces intense local heating and high pressures, with very short lifetimes.



Sun in the light of helium, SOHO

- If the bubble is hot enough, and the pressure high enough, fusion reactions like those that occur in the Sun could be produced within these tiny bubbles.

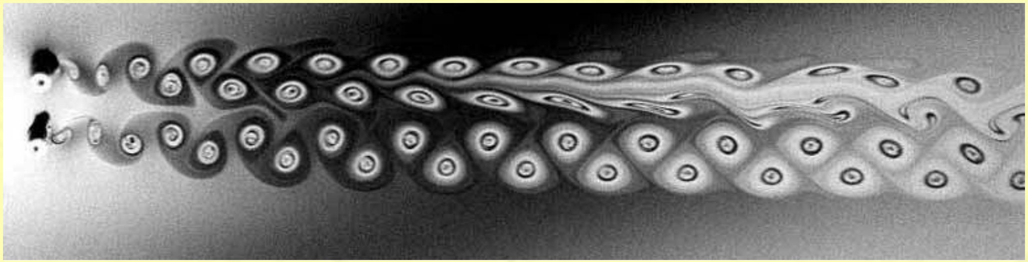
# Bubbles

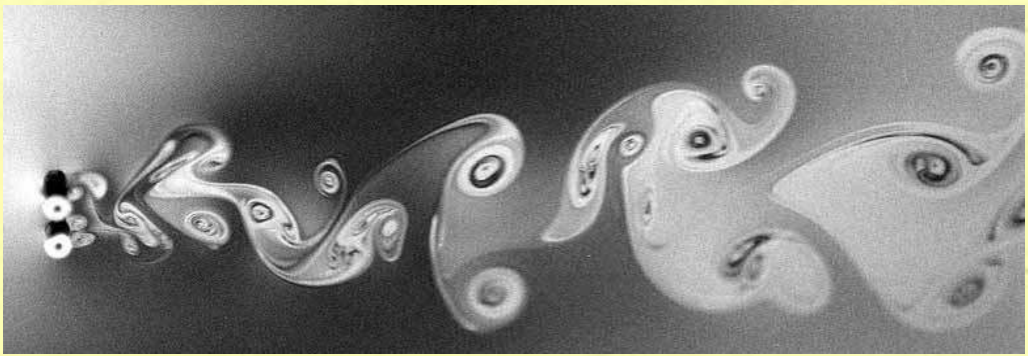
- Soap films are useful for studying fluid behaviors such as turbulence.

- Any object in the wind will leave a wake behind.
  One cylinder typically leaves a neatly arranged row of vortices.

# Bubbles

- If you ever hear a humming sound from a bicycle rack on your car's roof, you’re hearing the sound of many vortices of air separating in every second.





# Bubbles

- Some animals rely on bubbles to survive.

- When a snapping shrimp's claw snaps shut, a jet of water is forced out.

# Bubbles

- The reduced pressure inside the water jet allows the tiny air bubbles that naturally exist in seawater to expand rapidly.

- As the water pressure returns to its normal level, the air bubbles implode and generate a shock wave that is sufficient to stun or kill nearby prey.

# Bubbles

- Humans aren't the only ones to be fascinated by bubbles.

An adult male dolphin produces a ring and follows it up through the water.

# Bubbles

John Sullivan's 1991 tessellation of the 4D sphere by 120 regular dodecahedra. Three dodecahedra meet along each edge, at 120° angles, so the projection can be thought of as a cluster of soap bubbles.
This was the first rendering of soap bubbles made from the basic laws of optics for thin films.

# Bubbles

W. Brent Daniel and
Maarten Rutgers, 2002

# Bubble geometry

- The surfaces of bubbles and films are a very active field of geometry.

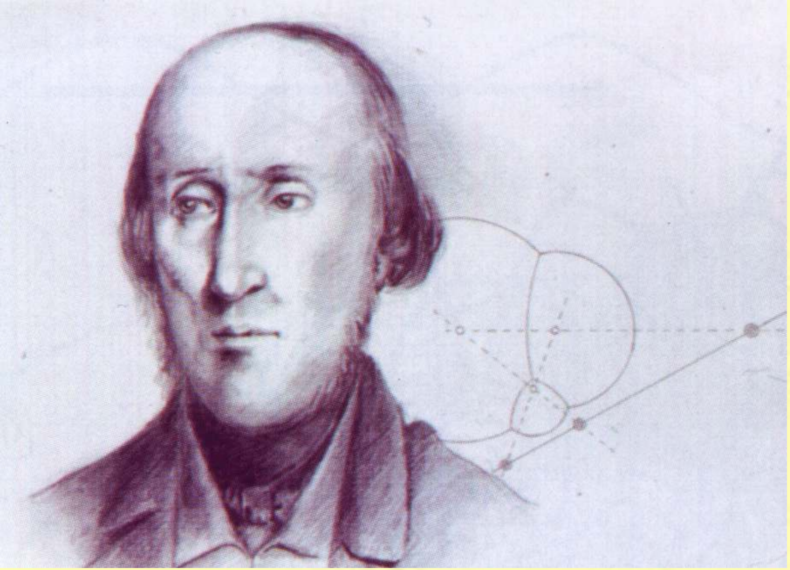
A Children Blowing Bubbles
1735, Jean-Etienne Liotard

# Bubble geometry

- The Belgian scientist Joseph Plateau went blind from staring at the sun for 30 seconds with his naked eyes while doing an experiment in his early twenties.

- He was aided in his later studies of soap films by his family and assistants.

# Bubble geometry



- He published his research in 1873, and since then many people have attempted to prove some of his surprising observations and hypotheses.

# Bubble geometry

- A froth of bubbles suggests that an infinite variety of configurations can be formed by joining various bubbles.

# Bubble geometry

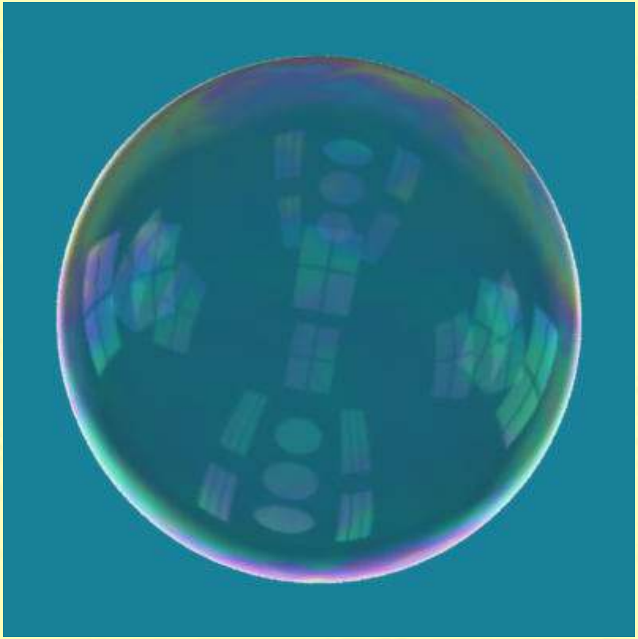
- Actually, bubble surfaces meet in only two ways!
  Three edges meet at a vertex with an angle of 120° between them, or
  four edges meet at a vertex with an angle of 109.47° between them.

# One bubble

- The most basic bubble shape, the sphere, has the smallest surface area for a given volume. Shapes that minimize surface area are called minimal surfaces.

- A soap bubble, like a balloon, is forced into its symmetric shape by the air contained inside being at a higher pressure than the air outside.

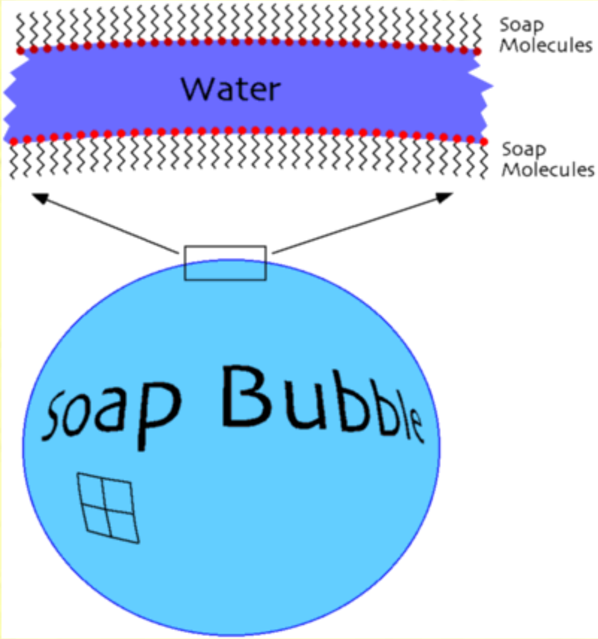
John M. Sullivan, 2000



# One bubble

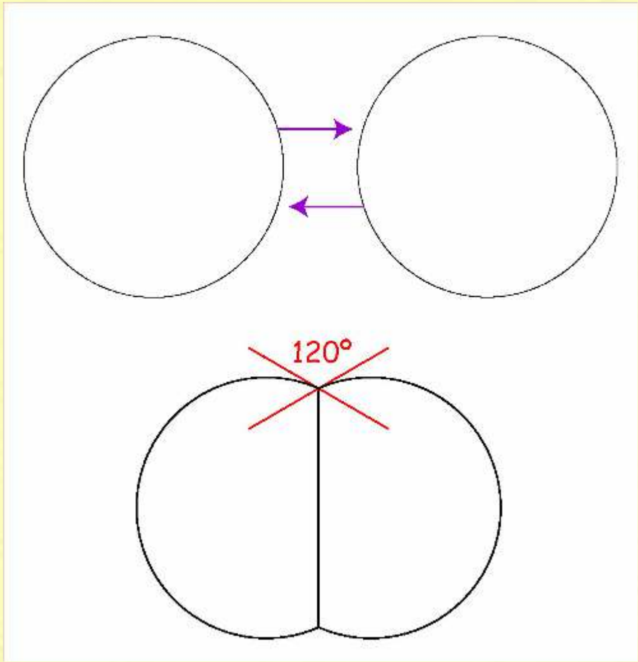
- The pressure on the film is given by the surface tension divided by the radius of the soap bubble.

Pressure = surface tension/radius



# Two bubbles

- Plateau observed that double bubbles always intersect at an angle of 120°.



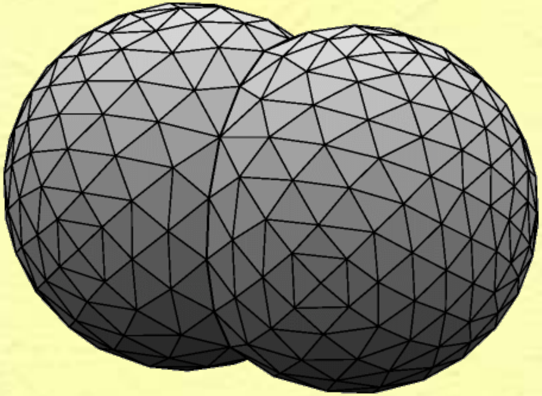

- If the two bubbles are exactly the same size, then the soap film wall separating them will be straight.

- He hypothesized that of all the possible shapes in the world, the double bubble is the most efficient at enclosing two equal volumes.

# Two bubbles

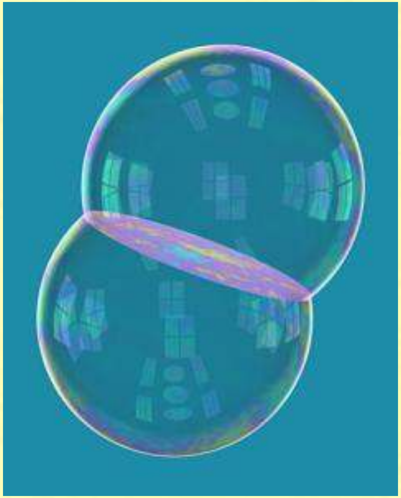
- It was known to the ancient Greeks that the sphere is the optimal shape to contain a given volume, although a proof had to wait until 1884.

- When one considers enclosing two separate volumes, the problem becomes considerably more difficult.



# Two bubbles

- Joel Hass at UC Davis and Roger Schlafly of RealSoftware stunned the mathematics community in 1995 by producing a rigorous proof that the double bubble is the minimal surface.

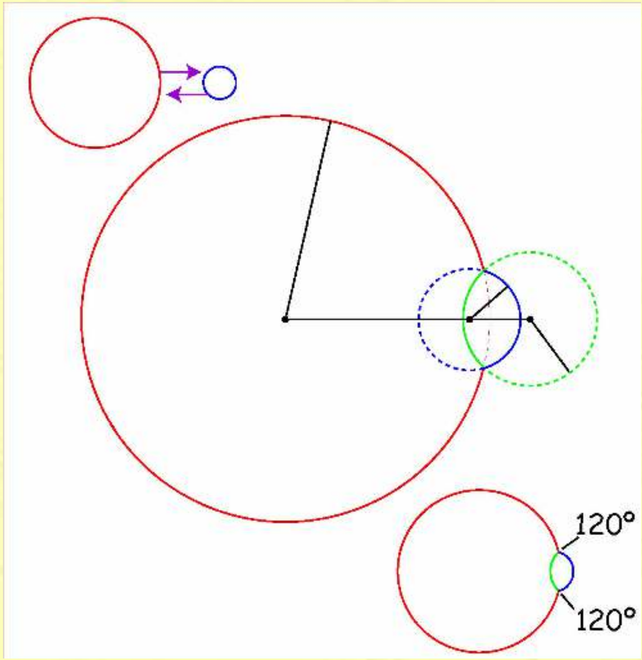


# Two bubbles

- If the two bubbles are not the same size, then the soap film wall separating them will be curved toward the larger bubble.

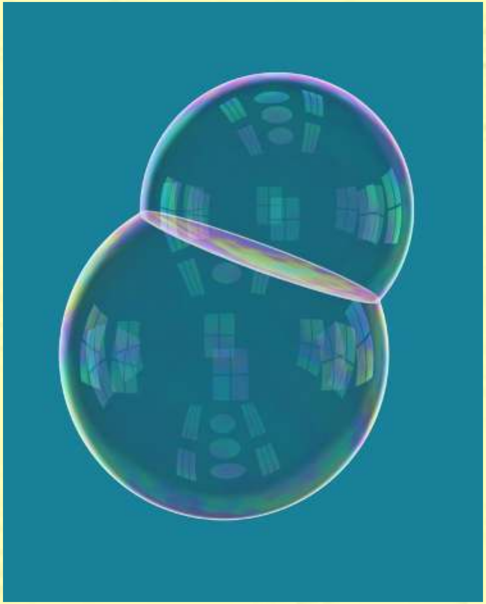
- This wall is a segment of a sphere whose center lies on the line through the centers of the two bubbles.

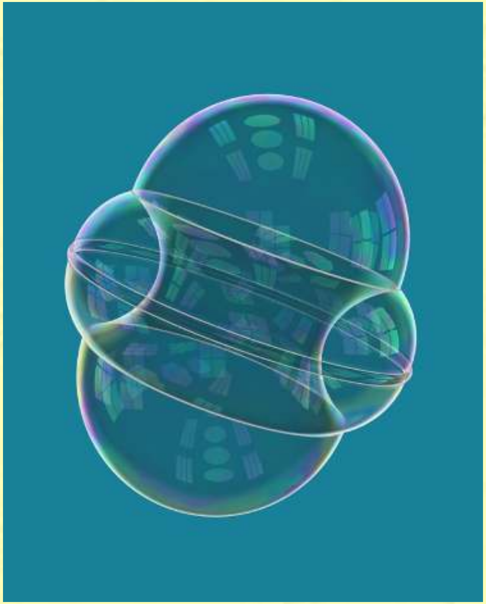
- The surface tension will be balanced at the 3 edges only if the angle is 120°.



# Two bubbles

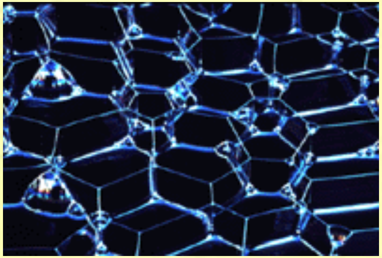
- In 2000, Michael Hutchings, Frank Morgan, Manuel Ritoré and Antonio Ros extended the minimal surface proof of double bubbles to include the case when the two volumes are not equal.





# Two bubbles

- If you take two sheets of plastic and blow bubbles between them you will notice that all of the vertices where three bubble walls meet form 120° angles.



- If your bubbles are of uniform size, you will notice that the cells form hexagons and start to look much like the cells of a beehive.

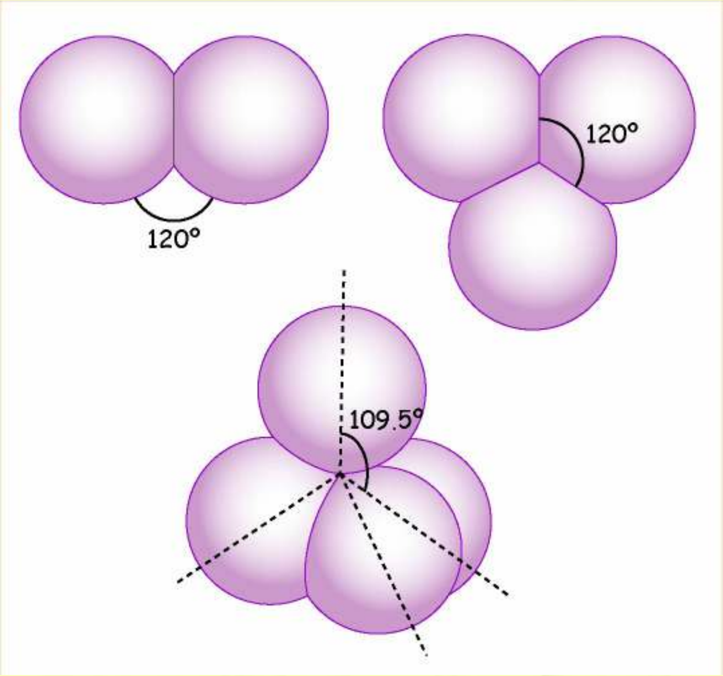
# Two bubbles

- Bees, like bubbles, try to be as efficient as possible when making the comb. They want to use the minimum possible amount of wax to get the job done. Hexagonal cells are the ticket.

# Four walls

- There is only other angle formed by thin films, 109.47°, and occurs when four walls intersect.

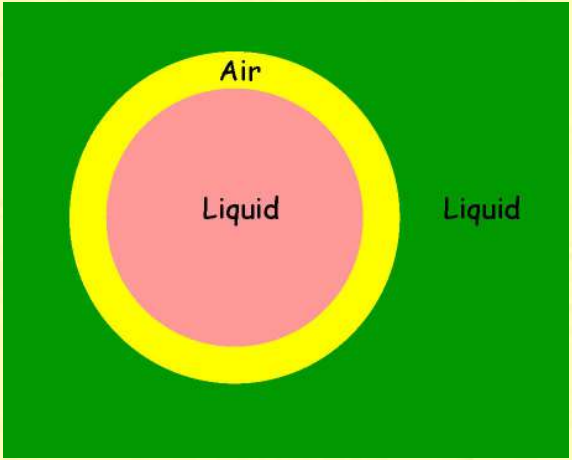
- You'll get a chance to explore both angles in today's in-class construction.



# Antibubbles

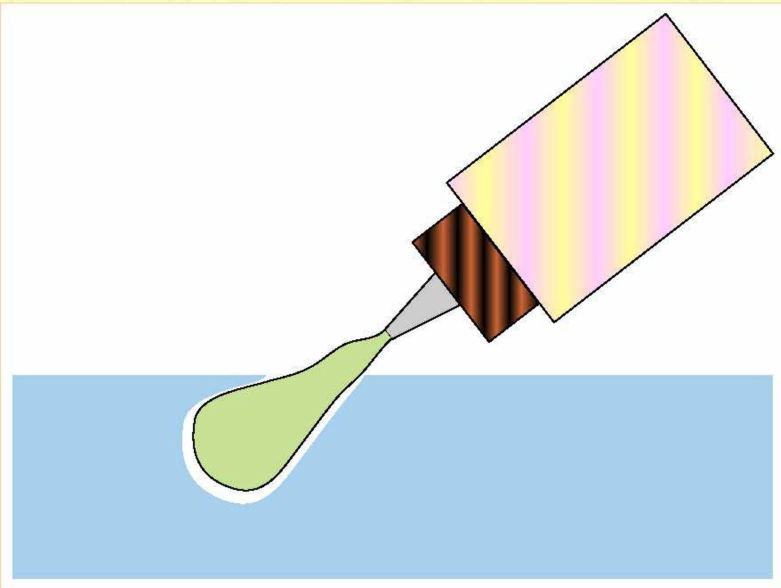
- A bubble in air is a thin film of liquid surrounding air.
  An antibubble in liquid is a thin film of air surrounding liquid.

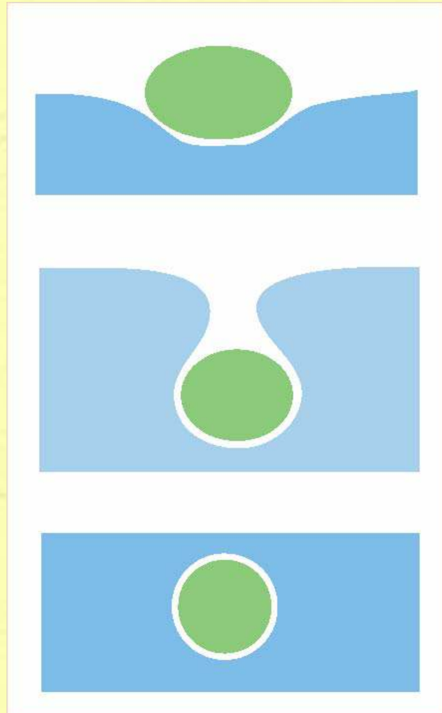
- An ordinary bubble is air inside and outside.
  An antibubble has liquid inside and outside.



# Antibubbles

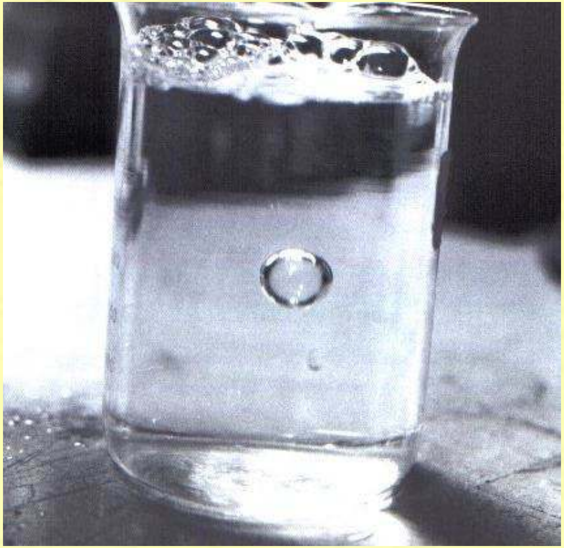
- Antibubbles form when a water droplet presses through a liquid surface with a thin coating of air.





# Antibubbles

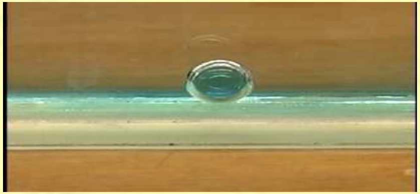
- If you add a little salt to the water in the bottle, your antibubbles will sink.



# Antibubbles

- If you put food coloring in the bottle with water, the antibubbles will be colored, which is a way to prove that antibubbles contain water, not air.

Movie of an antibubble with positive bouyancy, filled with a colored liquid.
Dave Hammond, 2002

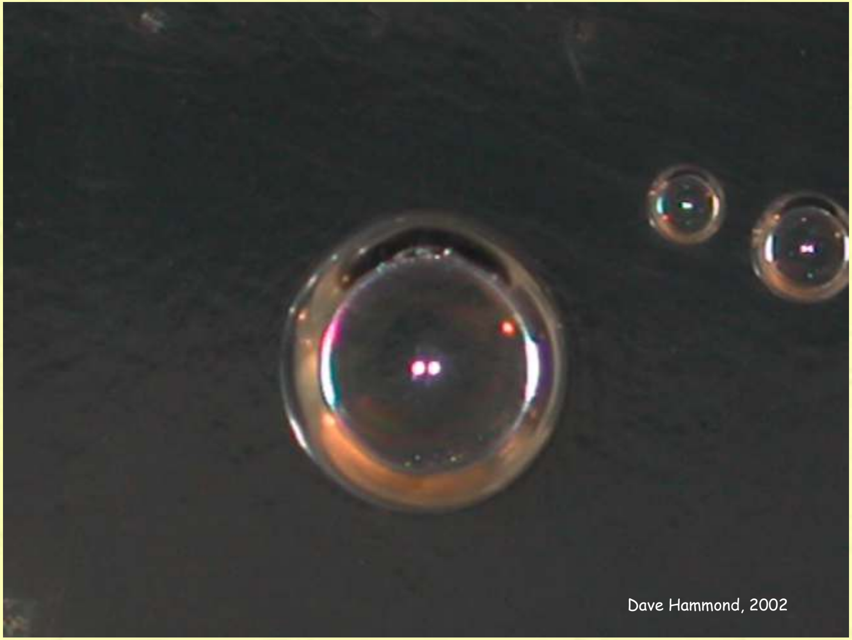


Movie of an anitbubble with negative bouyancy, filled with a colored liquid.  Notice air escaping when it bursts.
Dave Hammond, 2002

# Antibubbles

- Not a whole lot is known about antibubbles. How thick is the layer of air?

- How big or small can you make an antibubble?



# Antibubbles

- Some people might think antibubbles are pretty boring - that once the initial novelty has worn off there's not much to say about them.

# Antibubbles

- In energy plants, chemical or nuclear, the cooling water is carefully monitored.

- In some of these systems, sensors count the number of bubbles flowing through the cooling pipes.

H-1NF heliac

# Antibubbles

- From this, one gathers how much the cooling capacity has been degraded by air bubbles, and hence how to change the cooling water flow.

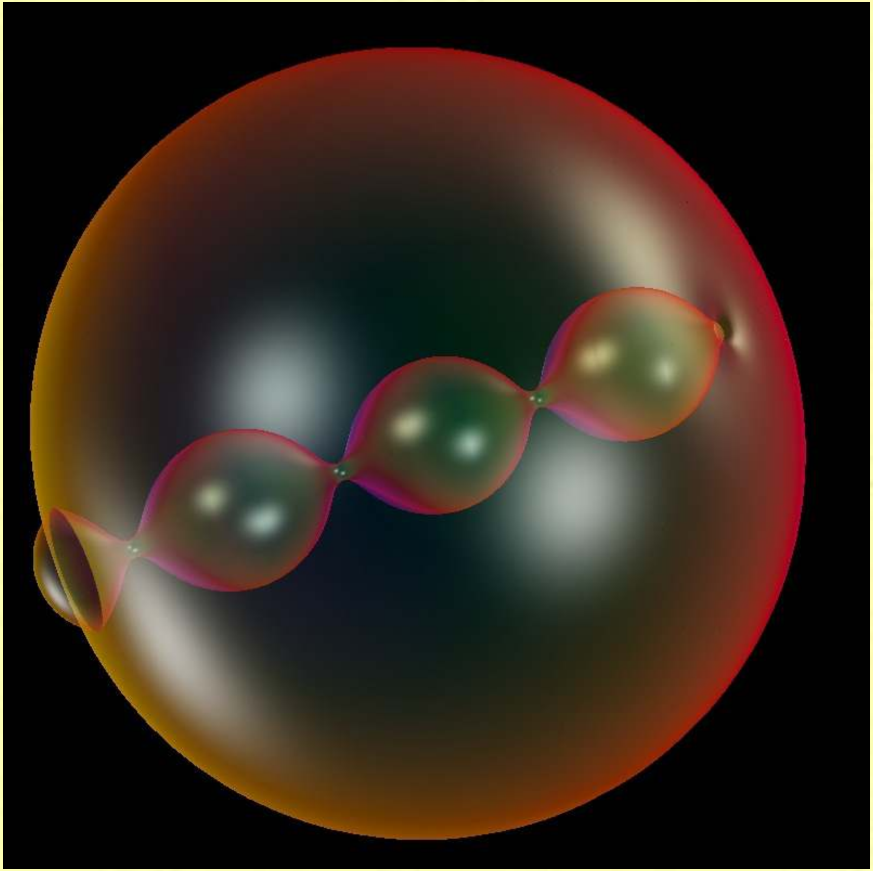
- If the bubbles are actually antibubbles, the cooling degradation will be overestimated.

# Antibubbles

- This may have adverse effects on the control system.

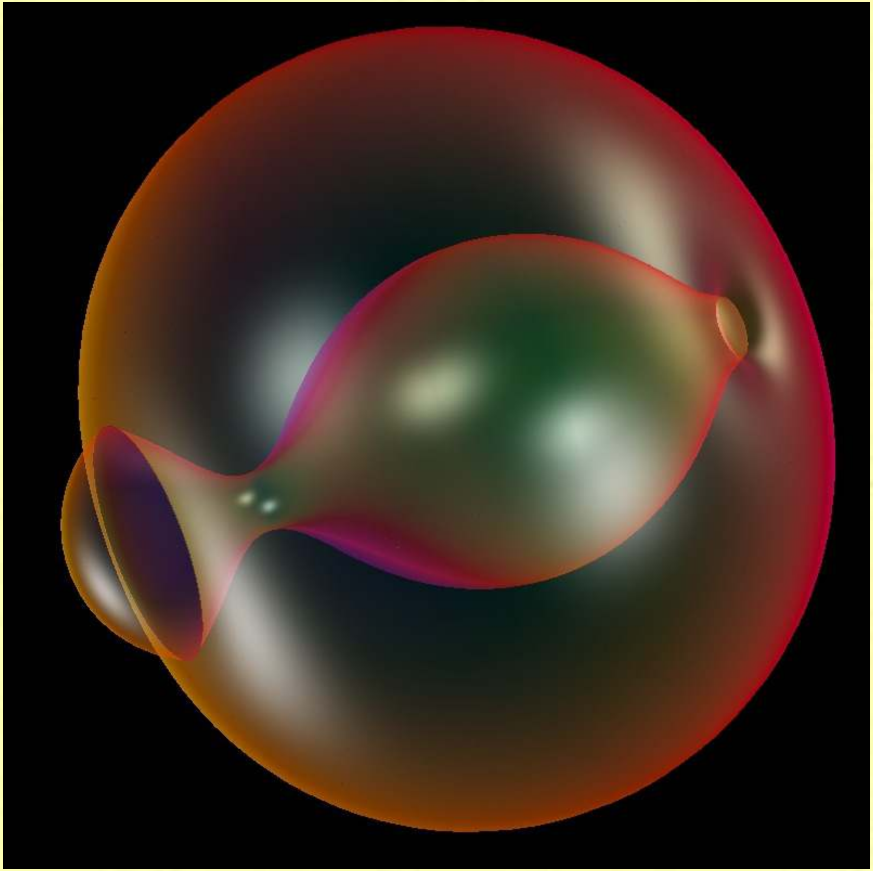
- In extreme cases, the control system could oscillate, causing catastrophic damage to the plant, or worse.

# Bubble mathart

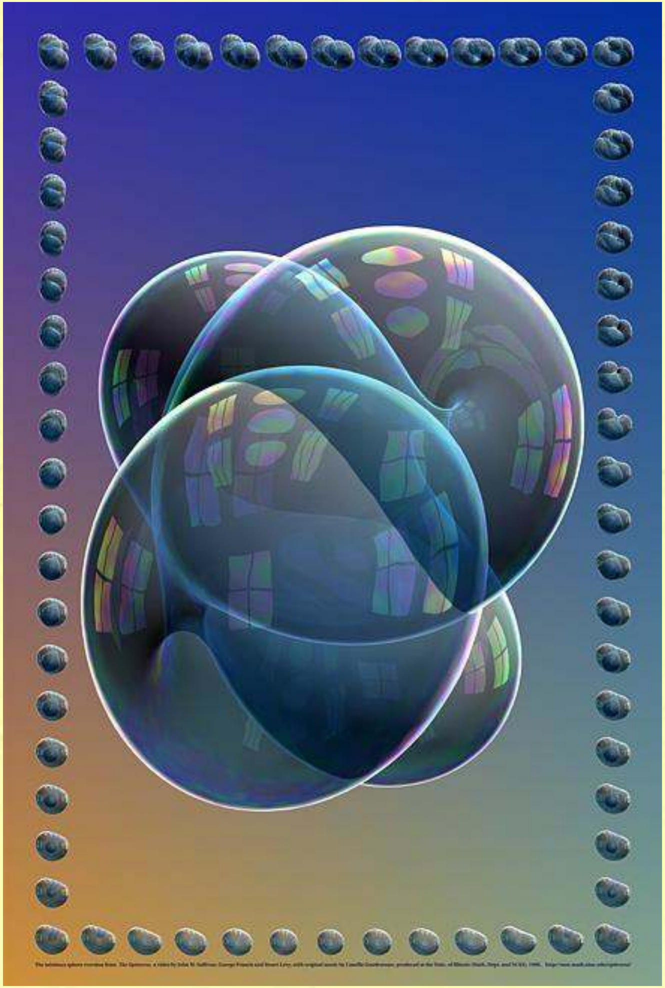


Bubmap3,
Jim Hoffman, 2000

# Bubble mathart

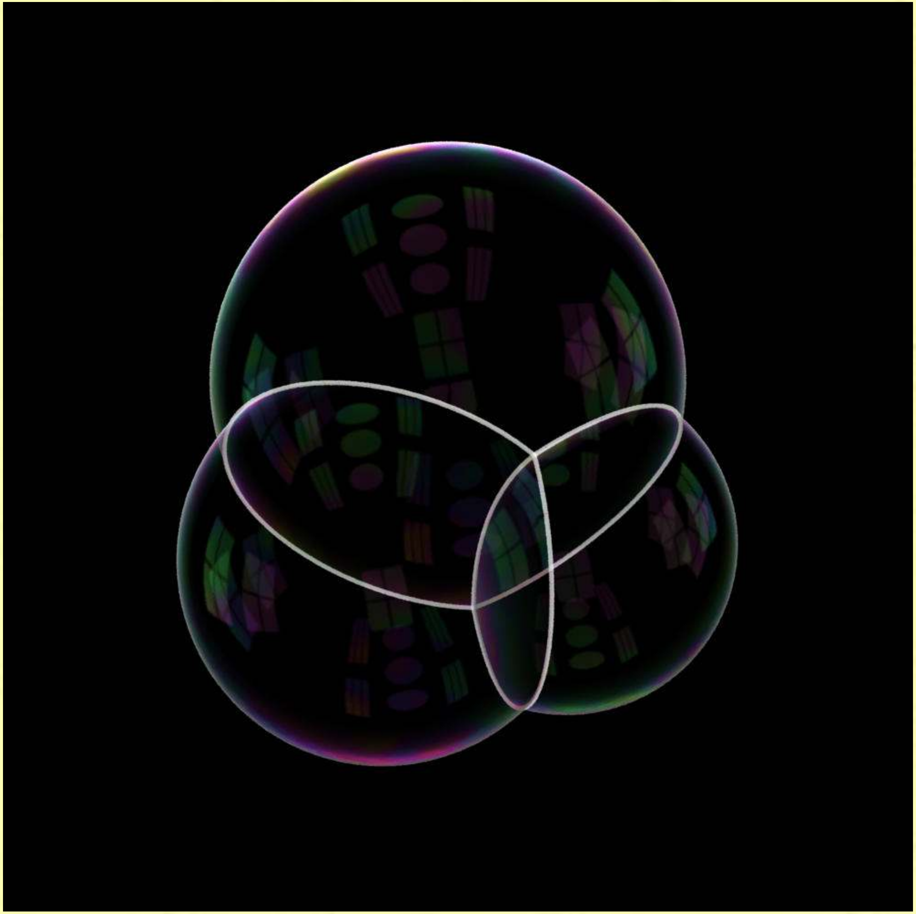


Bubmap1,
Jim Hoffman, 2000



Minmax Sphere,
John Sullivan, 2001

# Bubble mathart



Triple C,
John Sullivan, 2001

# Bubble mathart

Single vortex street
Marteen Rutgers, 2001

# Bubble mathart

- During today's in-class construction you'll discover some of the geometrical properties of bubble surfaces  and have a bit of fun making bubbles.