Clouds are not spheres, coastlines are not circles,
bark is not smooth, nor does lightning travel in straight lines.

Benoit B. Mandelbrot

School of the Art Institute of Chicago

Geometry of Art and Nature

Frank Timmes

ftimmes@artic.edu

flash.uchicago.edu/~fxt/class_pages/class_geom.shtml

Syllabus

1	Sept 03	Basics and Celtic Knots
2	Sept 10	Golden Ratio
3	Sept 17	Fibonacci and Phyllotaxis
4	Sept 24	Regular and Semiregular tilings
5	Oct 01	Irregular tilings
6	Oct 08	Rosette and Frieze groups
7	Oct 15	Wallpaper groups
8	Oct 22	Platonic solids
9	Oct 29	Archimedian solids
10	Nov 05	Non-Euclidean geometries
11	Nov 12	Bubbles
12	Dec 03	Fractals

Sites of the Week

- astronomy.swin.edu.au/~pbourke/fractals/
- usrwww.mpx.com.au/~peterstone/
- www.angelfire.com/art2/fractals/

Class #12

- Fractals
- Non-integer dimensions
- Iterations

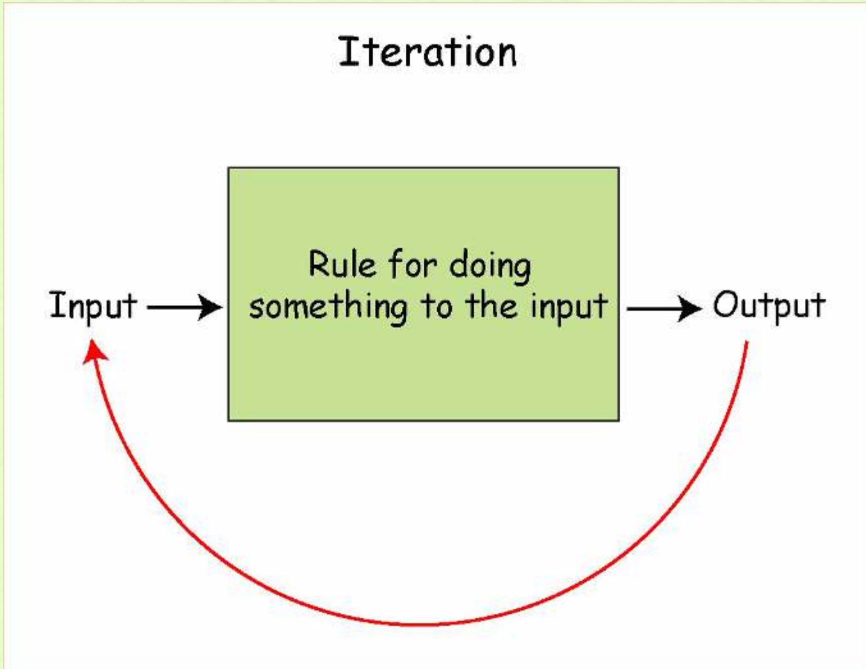
Fractals

- Where classical geometry deals with objects of integer dimensions, fractal geometry describes non-integer dimensions.

Benoit B. Mandelbrot
and Marcelo B. Ribeiro

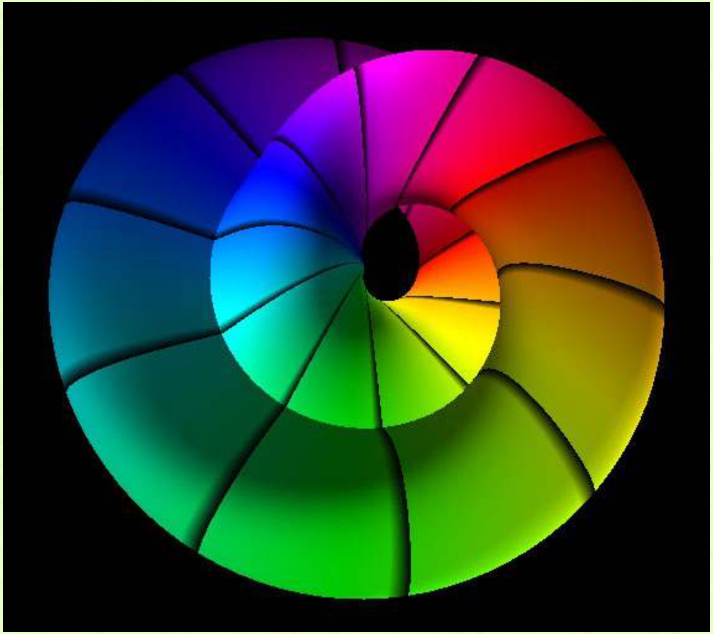
Fractals

- Non-integer dimensions can be made by picking a starting point, an iteration rule, and the number of iterations to do.



Not an integer

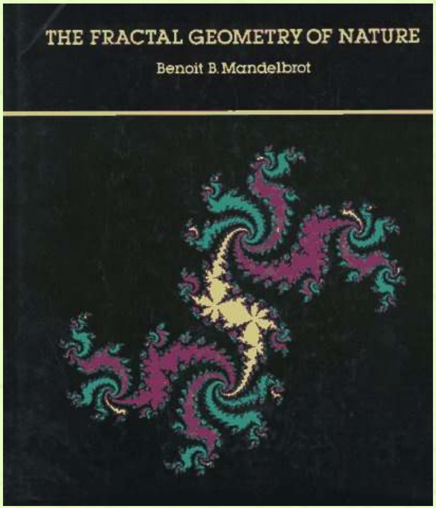
- Zero dimensional points, one dimensional lines and curves, two dimensional plane figures like squares and circles, and three dimensional solids such as cubes and spheres make up the world as we have previously understood it.



Geometry
1999, Paul Bourke

Not an integer

- However, many natural phenomena are better described with a dimension part way between two whole numbers.

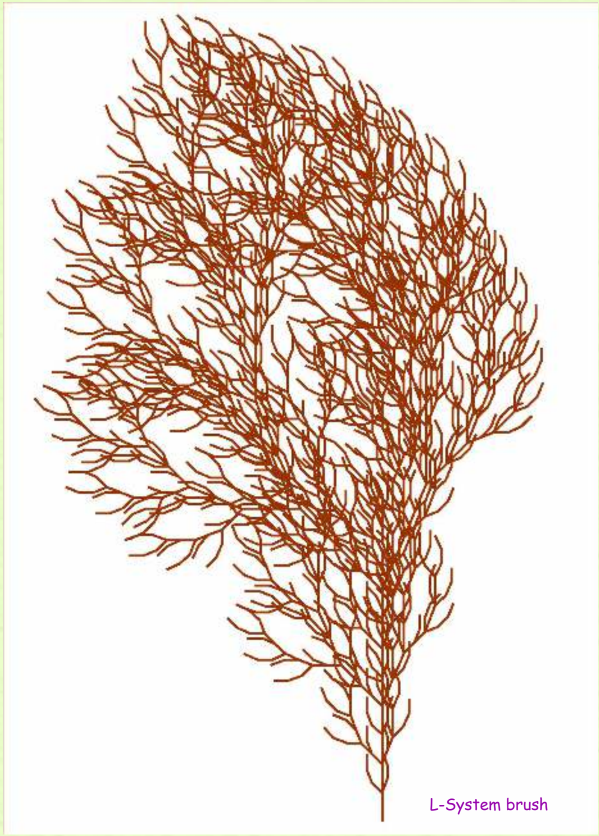


Leafifs
1999, Paul Bourke

Not an integer

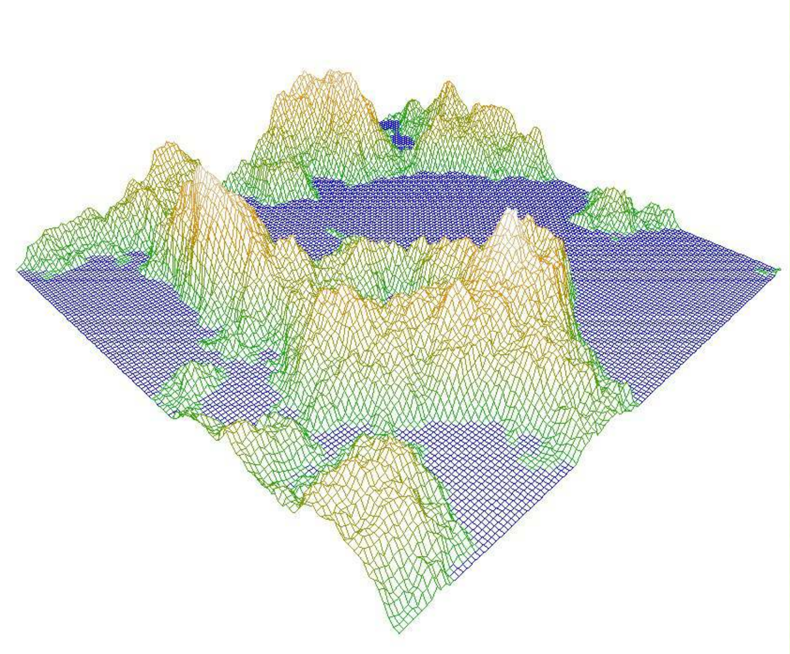
- A straight line has a dimension of one, but a fractal curve will have a dimension between one and two depending on how much area it takes up as it twists and curves.

- The more the fractal fills a plane, the closer it approaches two dimensions.



Not an integer

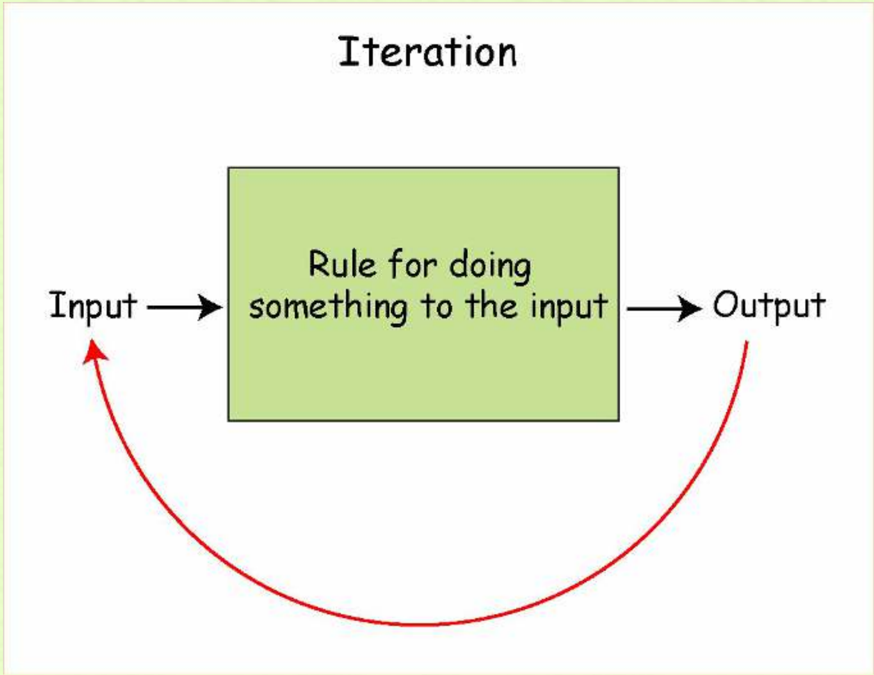
- Likewise, a "hilly fractal scene" has a dimension between two and three.



- A landscape made up of a large hill covered with tiny bumps would be close to the second dimension, while a rough surface composed of many medium-sized hills would be close to the third dimension.

Fractals

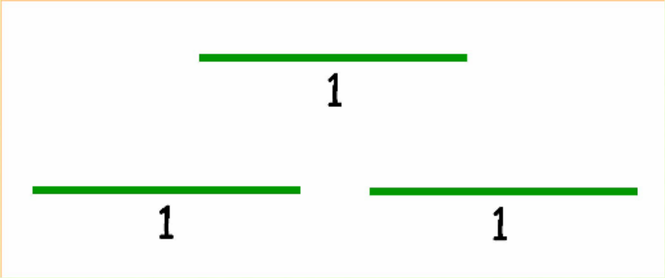
- Non-integer dimensions can be made by picking a starting point, an iteration rule, and the number of iterations to do.



Not an integer

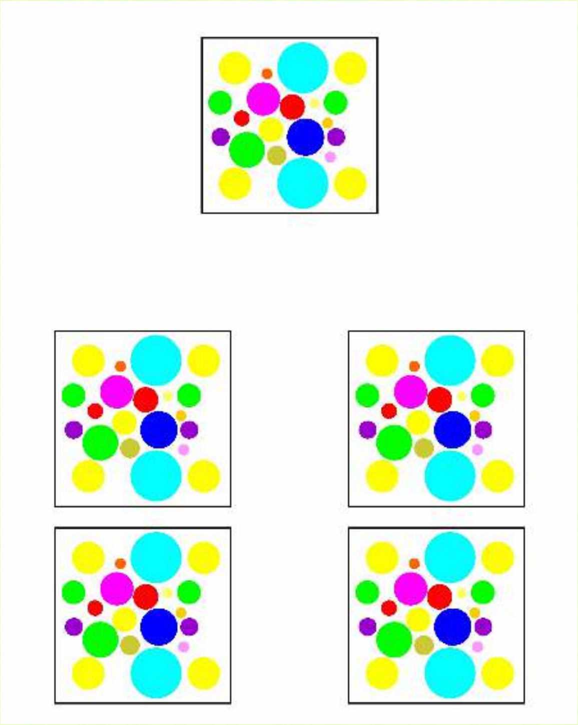
- Let's look further at what we mean by dimension.

- Take a self-similar figure like a line segment, and double its length. Doubling the length gives two copies of the original segment.



Not an integer

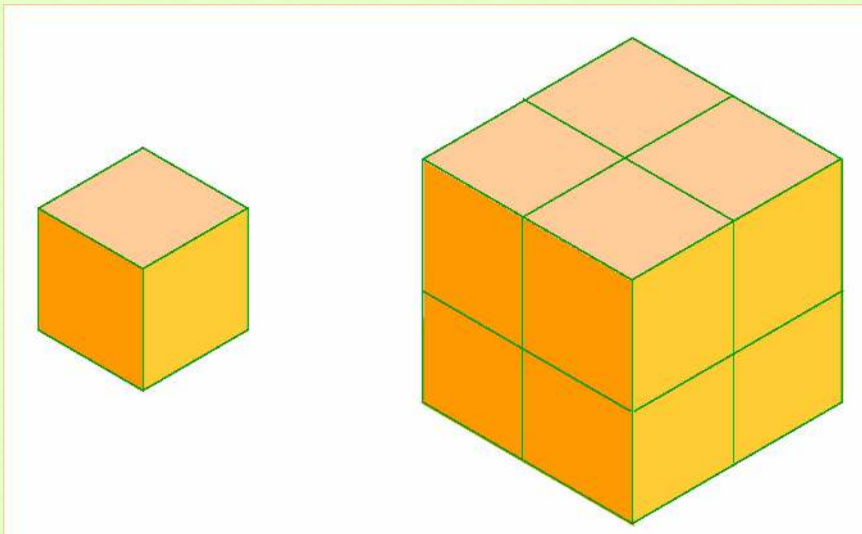
- Take another self-similar figure, this time a square 1 unit by 1 unit.
  Now multiply the length and width by two.



- How many copies of the original size square do you get? Doubling the sides gives four copies.

Not an integer

- Take a 1 by 1 by 1 cube and double its length, width, and height.
- How many copies of the original size cube do you get?
  Doubling the sides gives eight copies.

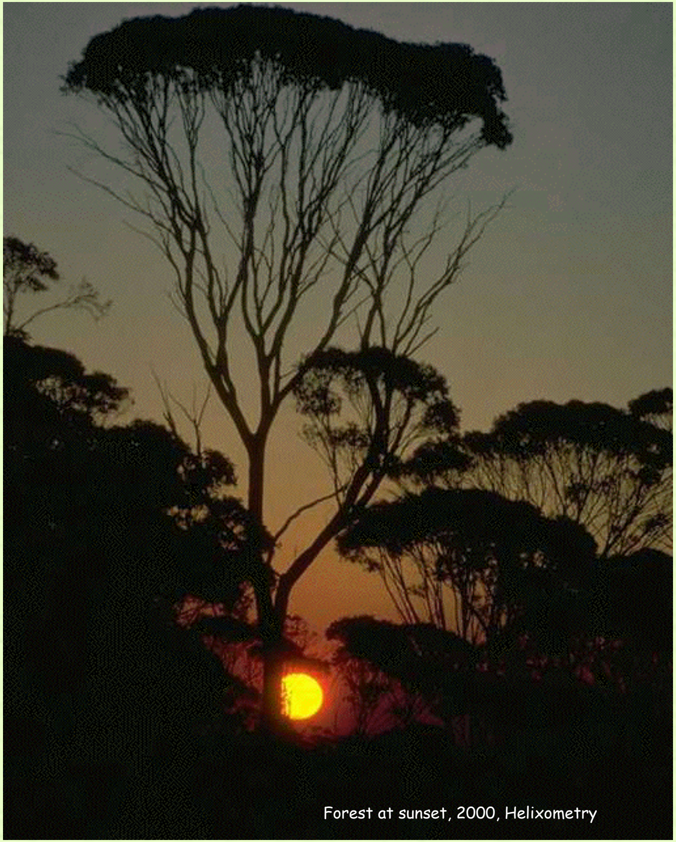


Not an integer

- Let's organize our information:

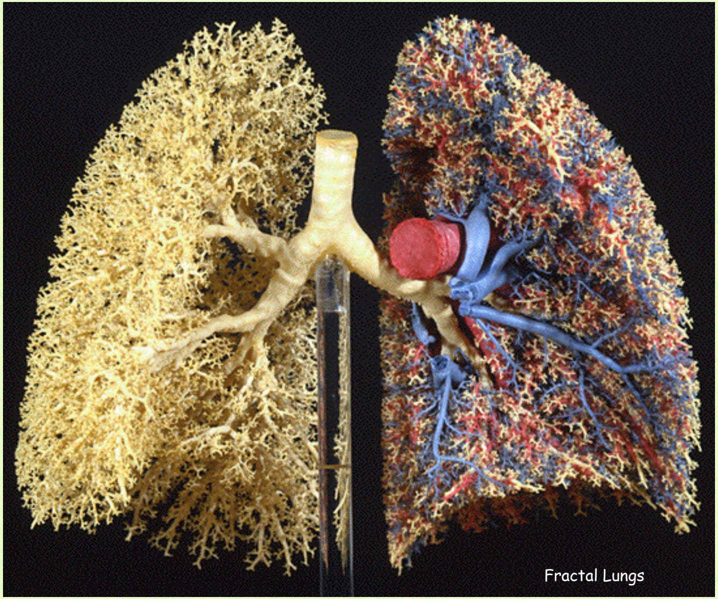
Figure	Dimension	Copies
Line	1	$2 = 2^1$
Square	2	$4 = 2^2$
Cube	3	$8 = 2^3$

- Do you see a pattern?



Not an integer

- It appears that the dimension is the exponent - and it is!



- When we double the sides and get a similar figure, we write the number of copies as a power of two and the exponent will be the dimension.

Not an integer

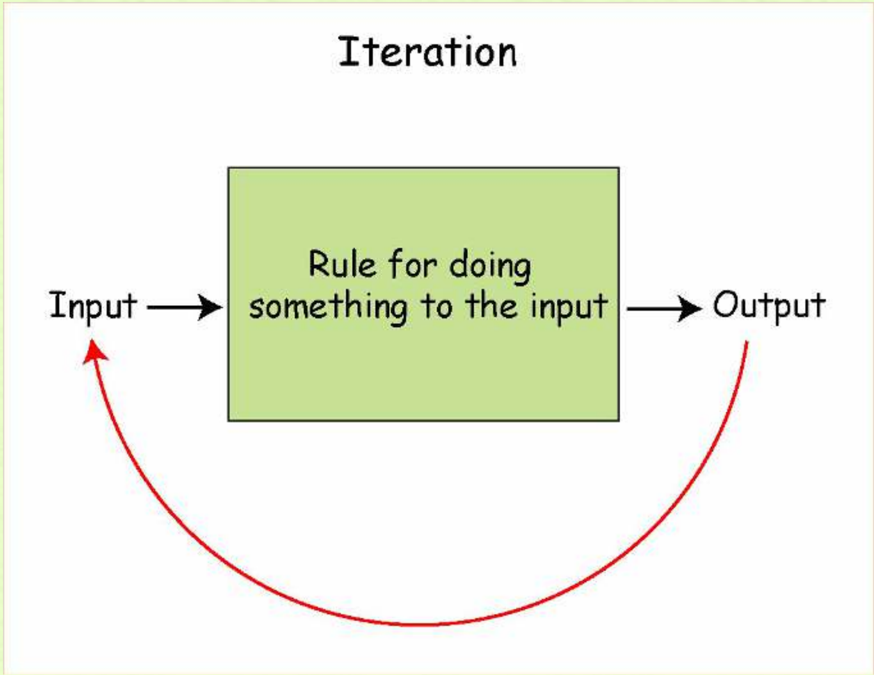
- Let's add that general case as a row to the table.

Figure	Dimension	Copies
Line	1	$2 = 2^1$
Square	2	$4 = 2^2$
Cube	3	$8 = 2^3$
Doubling	d	$n = 2^d$

- We'll see what the dimension of some fractals are as we look into making them.

Fractals

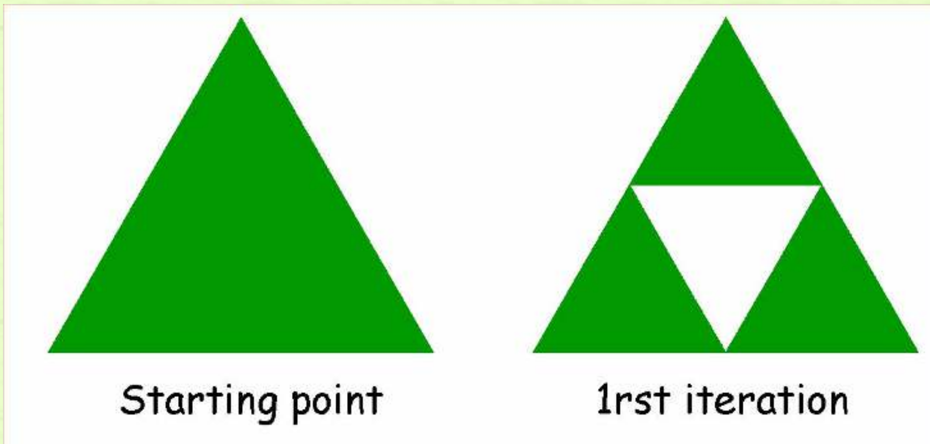
- Non-integer dimensions can be made by picking a starting point, an iteration rule, and the number of iterations to do.



Iteration

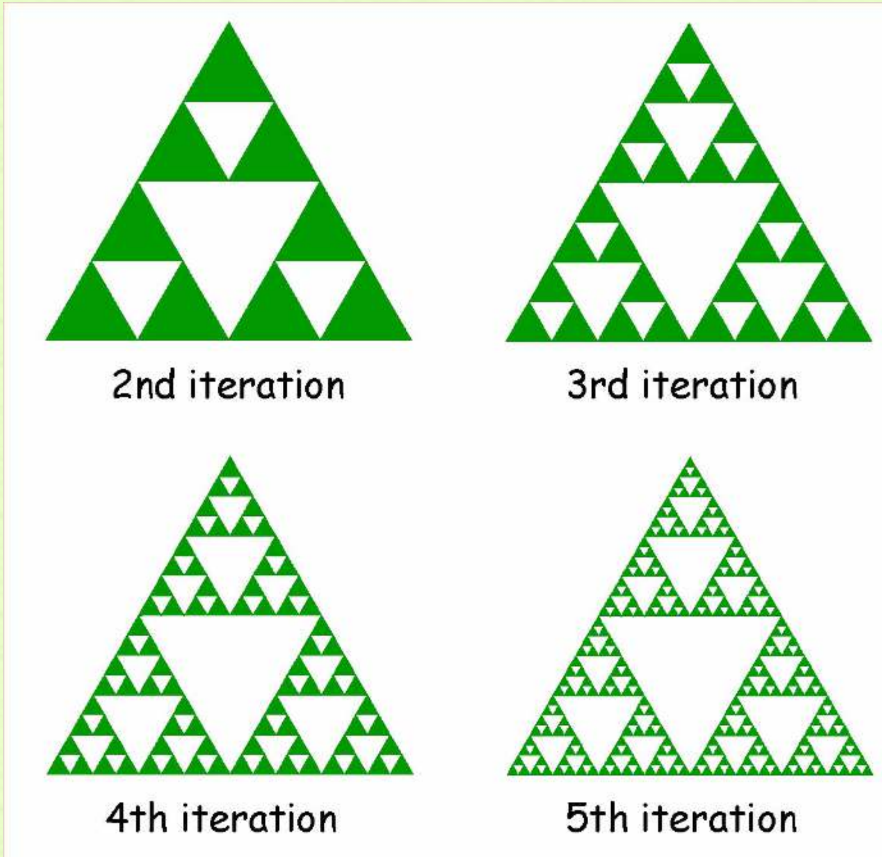
- Recall that to create a fractal we need to make up a rule, specify a starting point, and the number of iterations to do.

- Starting point: An equilateral triangle
  Rule: Connect the midpoints and cut out the middle
  Iterations: 5



Iteration

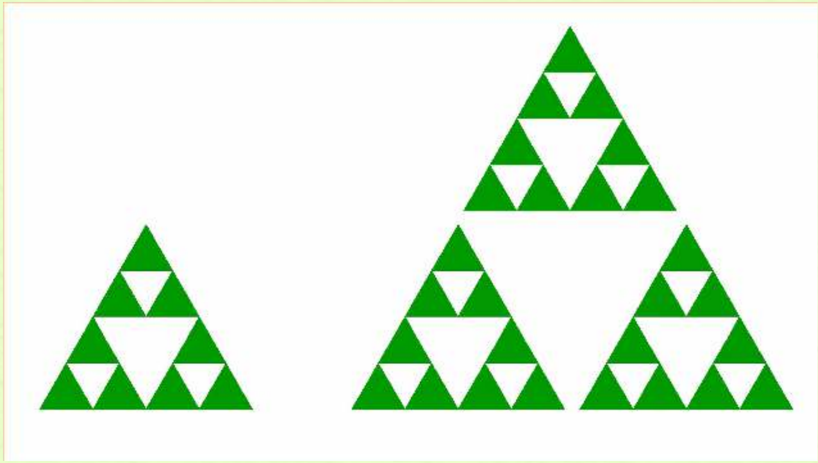
- This fractal is called a Sierpinski Triangle.



Iteration

- Start with a Sierpinski triangle and double the length of the sides.

- How many copies of the original triangle do we have?
  Remember that the white triangles are holes, so we don't count them.



Iteration

- Doubling the sides gives us three copies, so using our relation between the number of copies and dimension gives us $3 = 2^{d}$.

- So the fractal dimension of a Sierpinski triangle is d = 1.5894.

Iteration

Stage-4 Sierpinski tetrahedron ion the limb of a River Bushwillow
2003 Gayla Chandler

Iteration

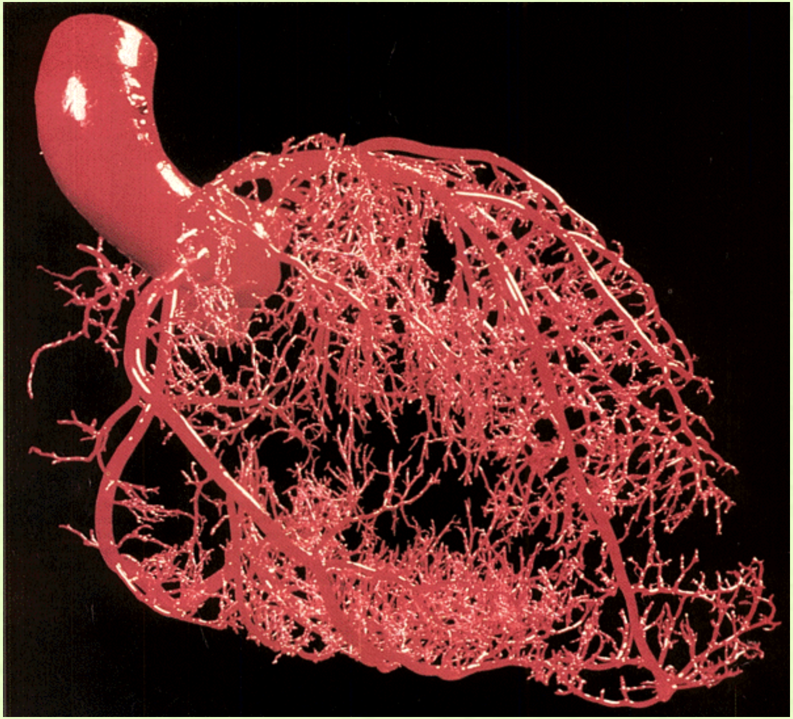
Stage-3 Sierpinski and a birds nest

2003 Gayla Chandler

Iteration

Menger Sponge
2001, Angelo Pesce

Interlude

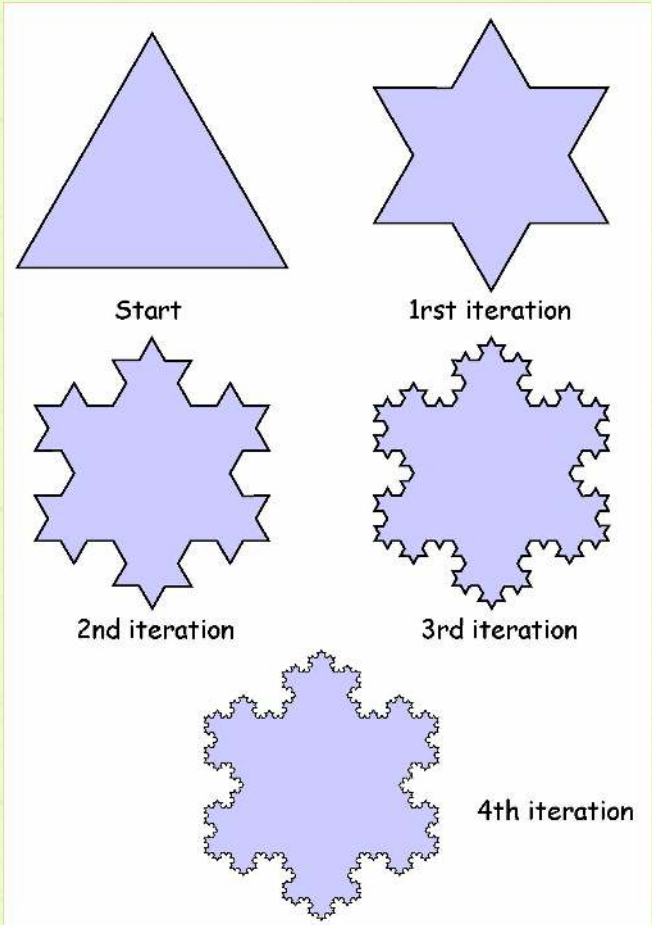


Iteration

- Start: An equilateral triangle

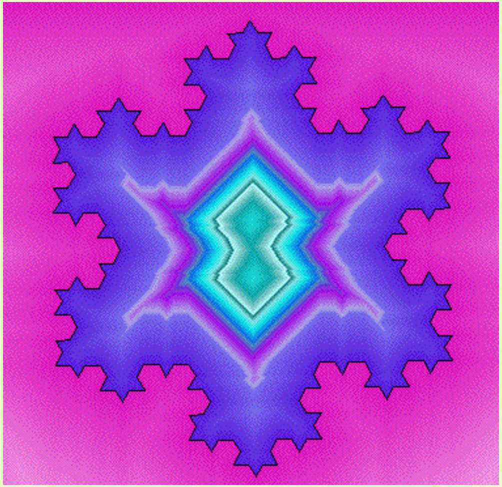
- Rule: Remove the middle third of any line, and replace it with two lines that each have the same length (1/3) as the remaining lines on each side.

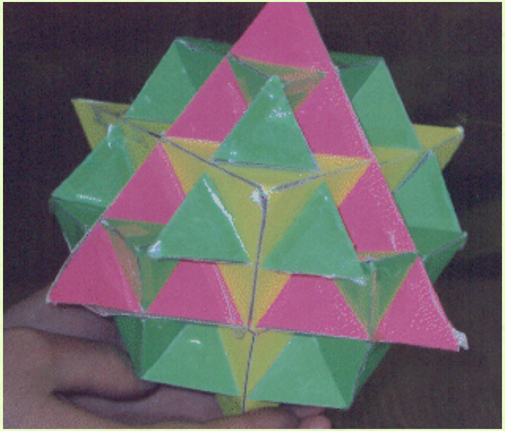
- Iterations: 4



Iteration

- This fractal is called a Koch snowflake, and has a dimension of log 4/log 3 = 1.261.





Iteration

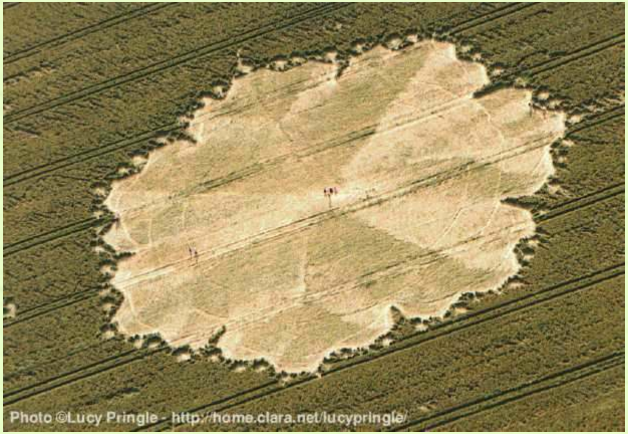
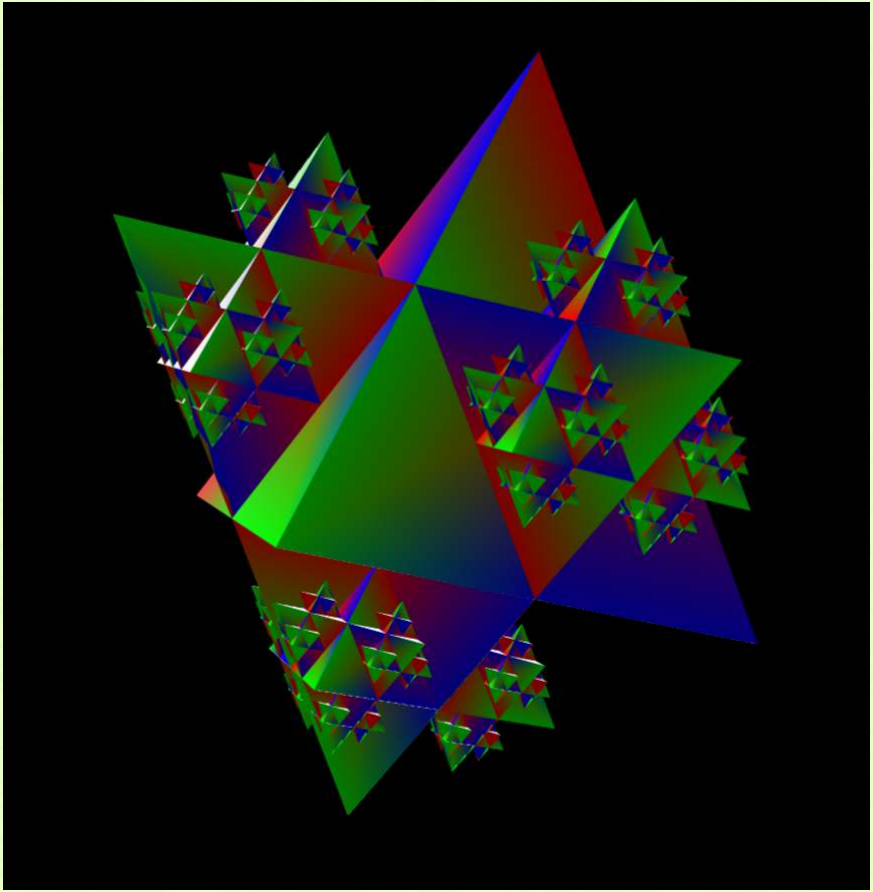


Photo ©Lucy Pringle - http://home.clara.net/lucypringle/

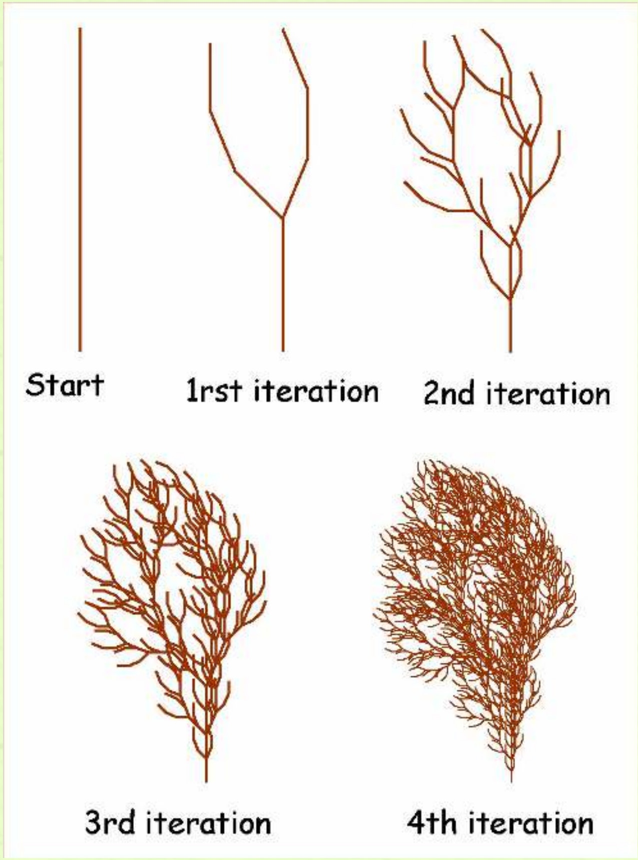

Photo ©Lucy Pringle

Iteration

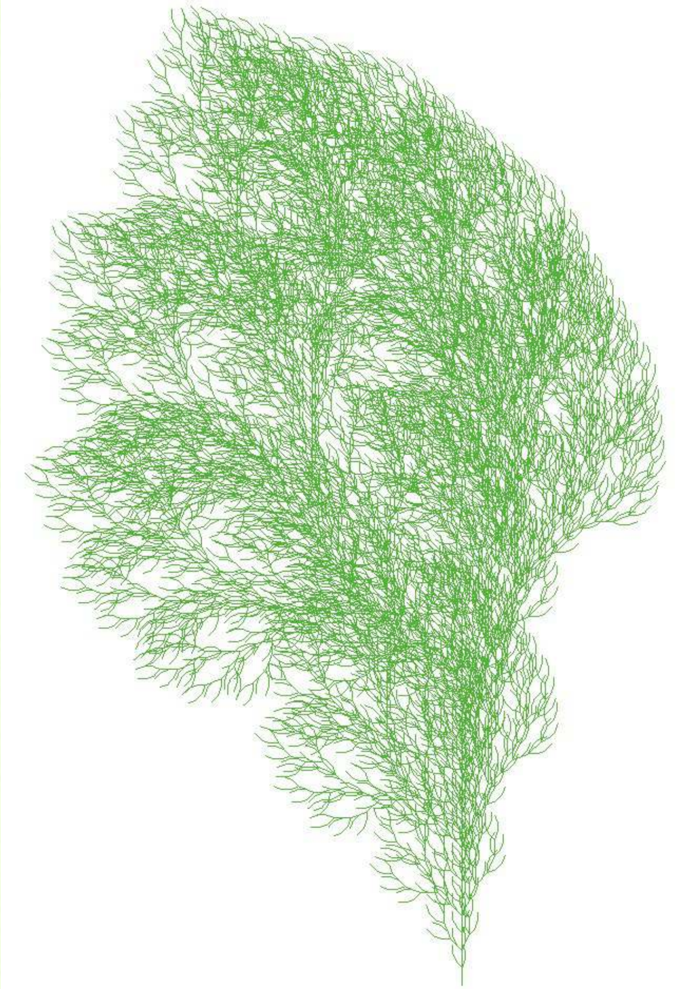


Plants

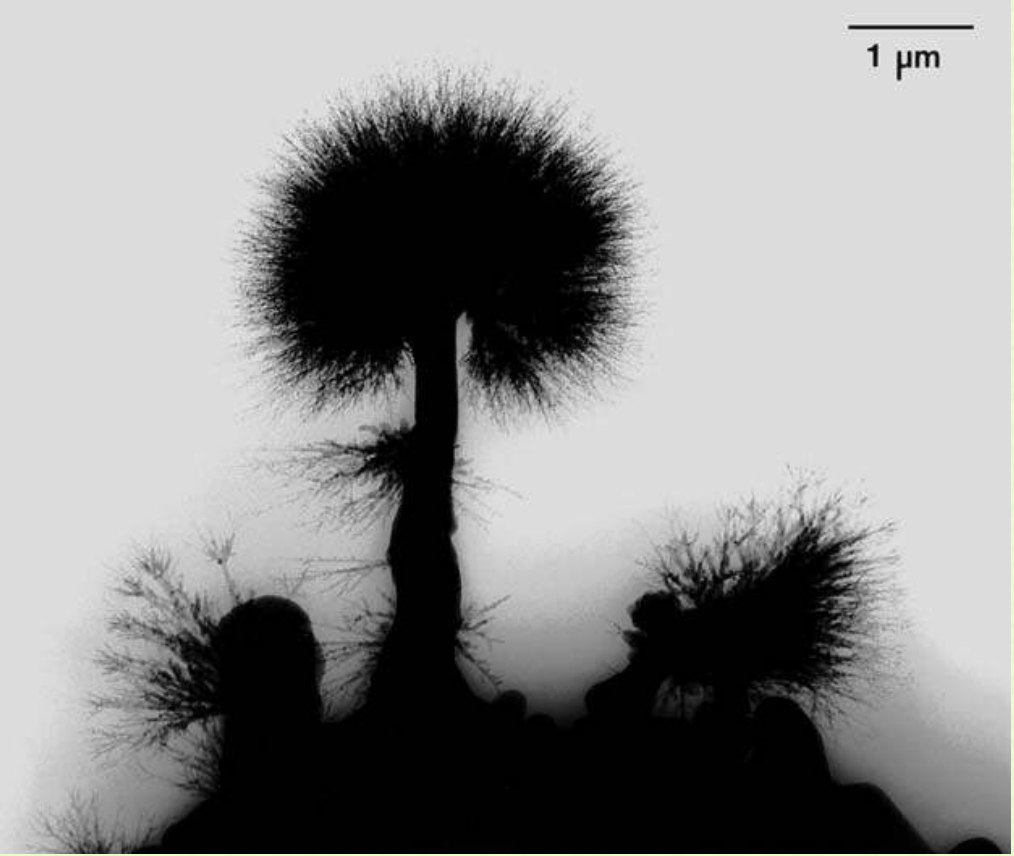
- Start: An equilateral triangle
- Can you see the rule visually at the first iteration?
- Iterations: 5



- Iteration five of the “windy bush”.



Plants



Plants

Plants

Plants

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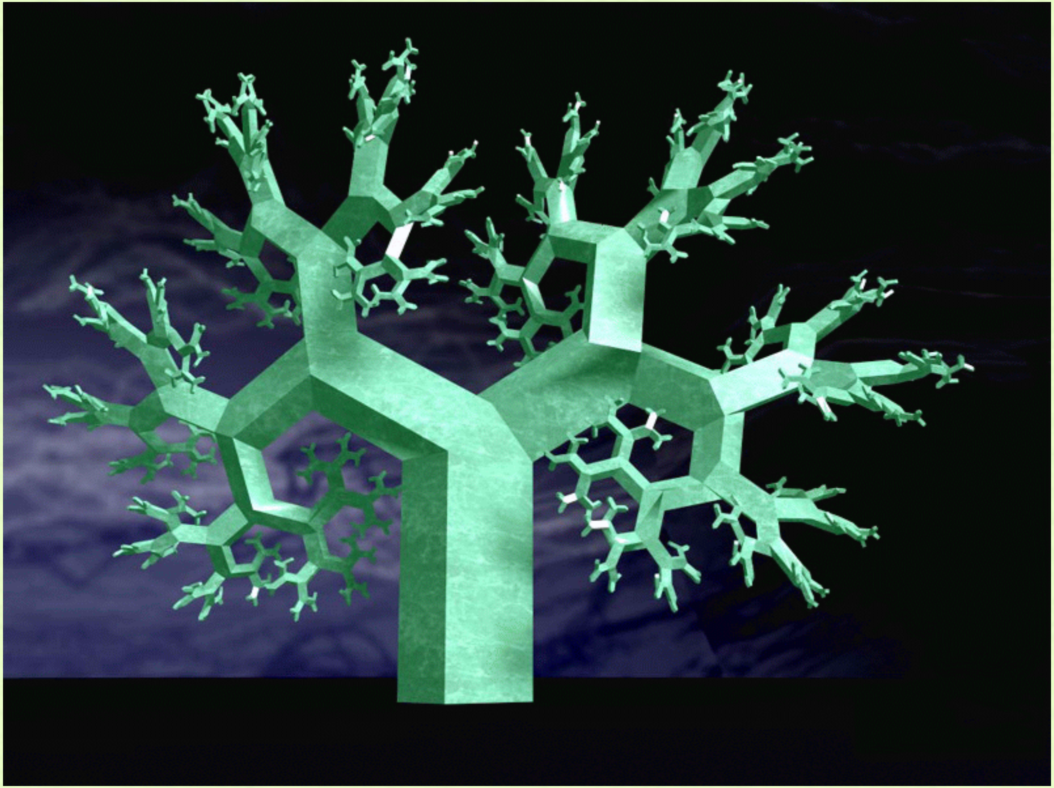


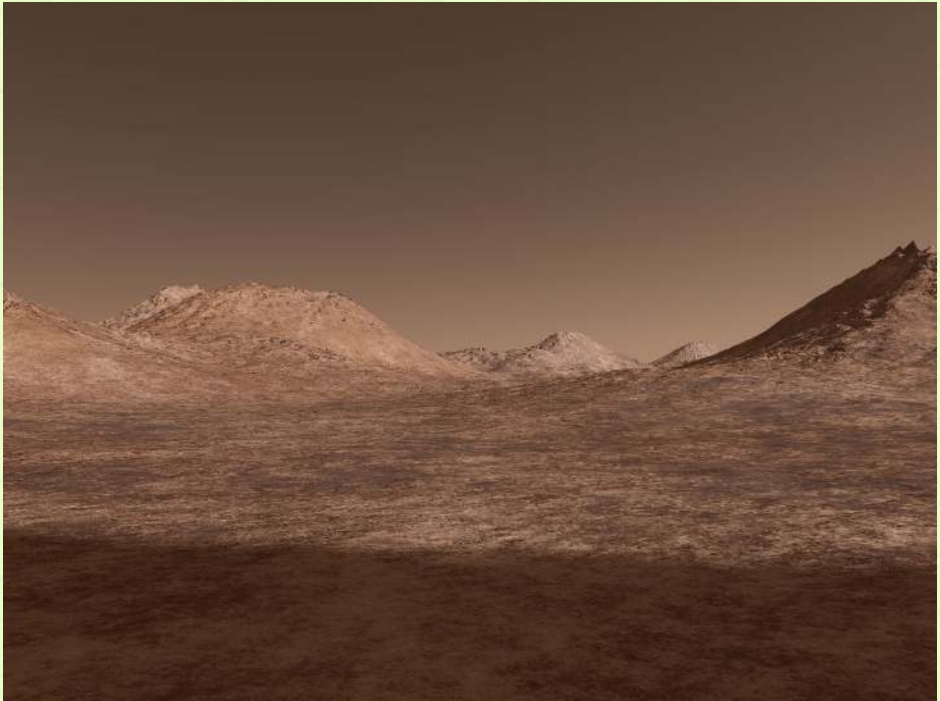
Landforms

- Fractal landscapes are often generated using “spatial subdivision”.

Adam Brown, 2002

Landforms

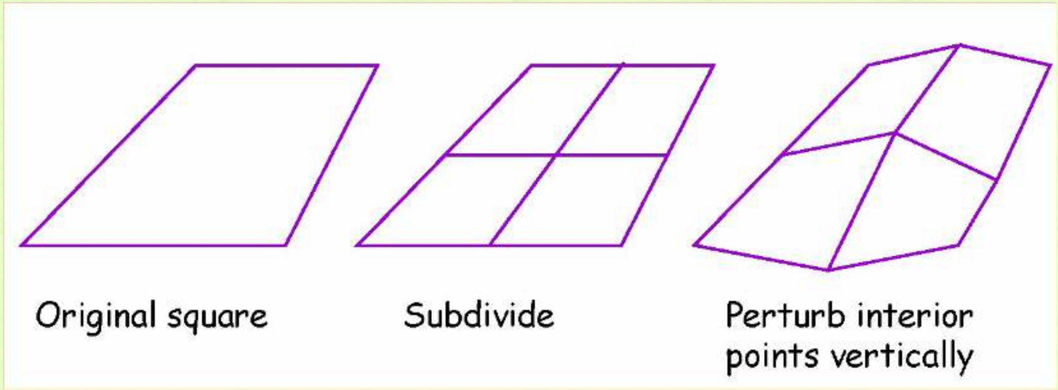
- For mathemagical reasons this yields surfaces that are strikingly similar in appearance to planetary terrains.



Adam Brown, 2002

Landforms

- The idea behind spatial subdivision is quite simple. Consider a square:



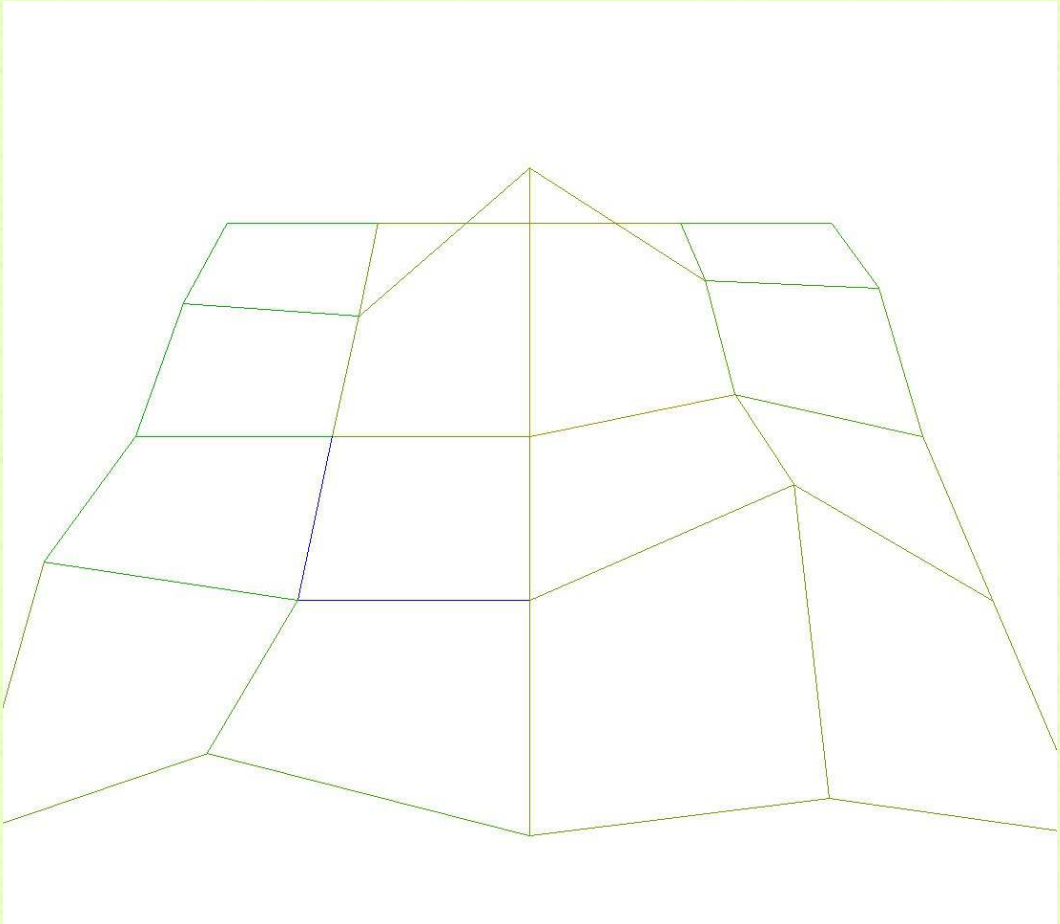
1) split the square up into a 2x2 grid

2) vertically perturb each of the 5 new vertices by a random amount

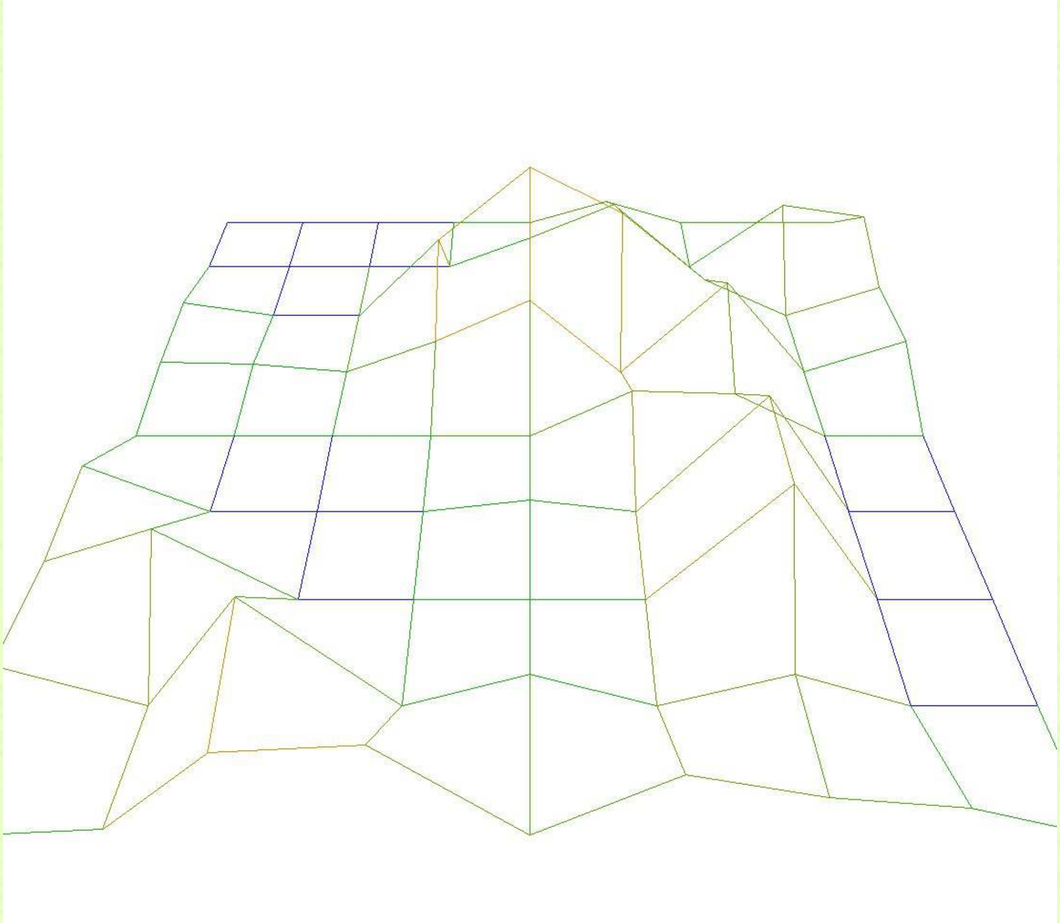
3) Iterate on each new square decreasing the perturbation each step

Landforms

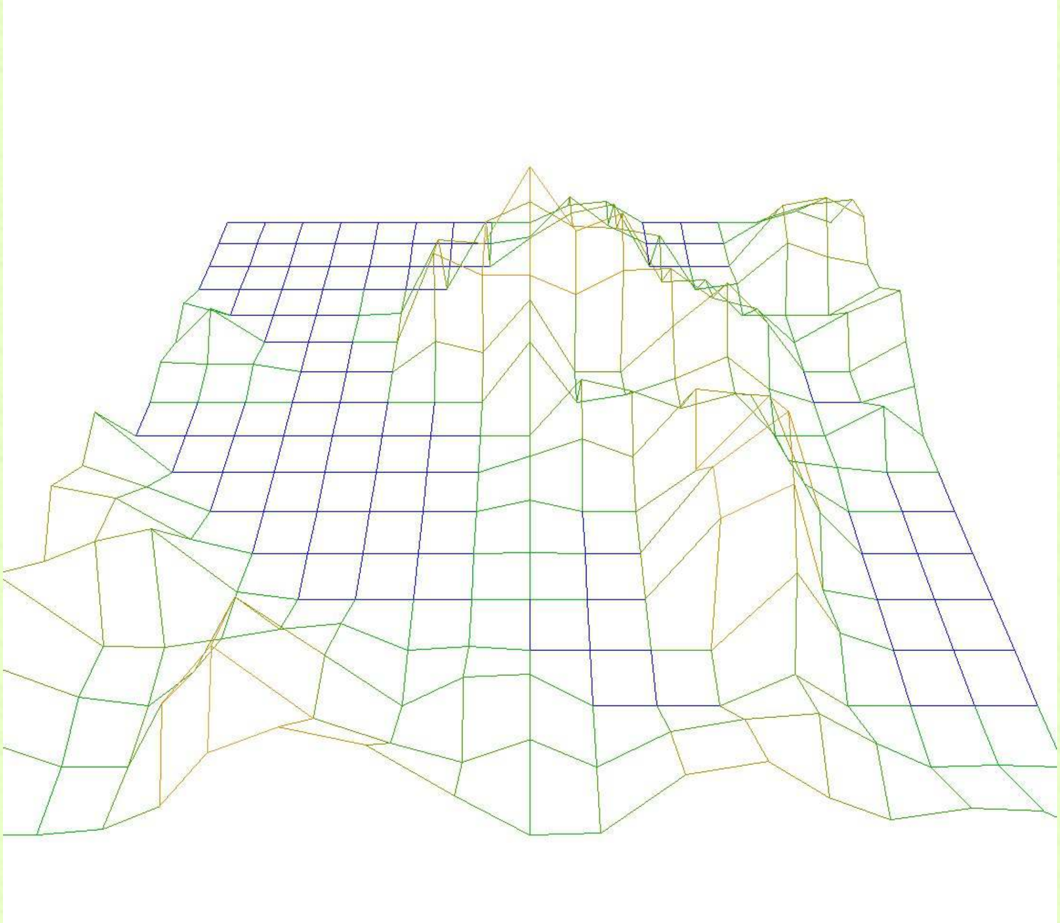
Landforms



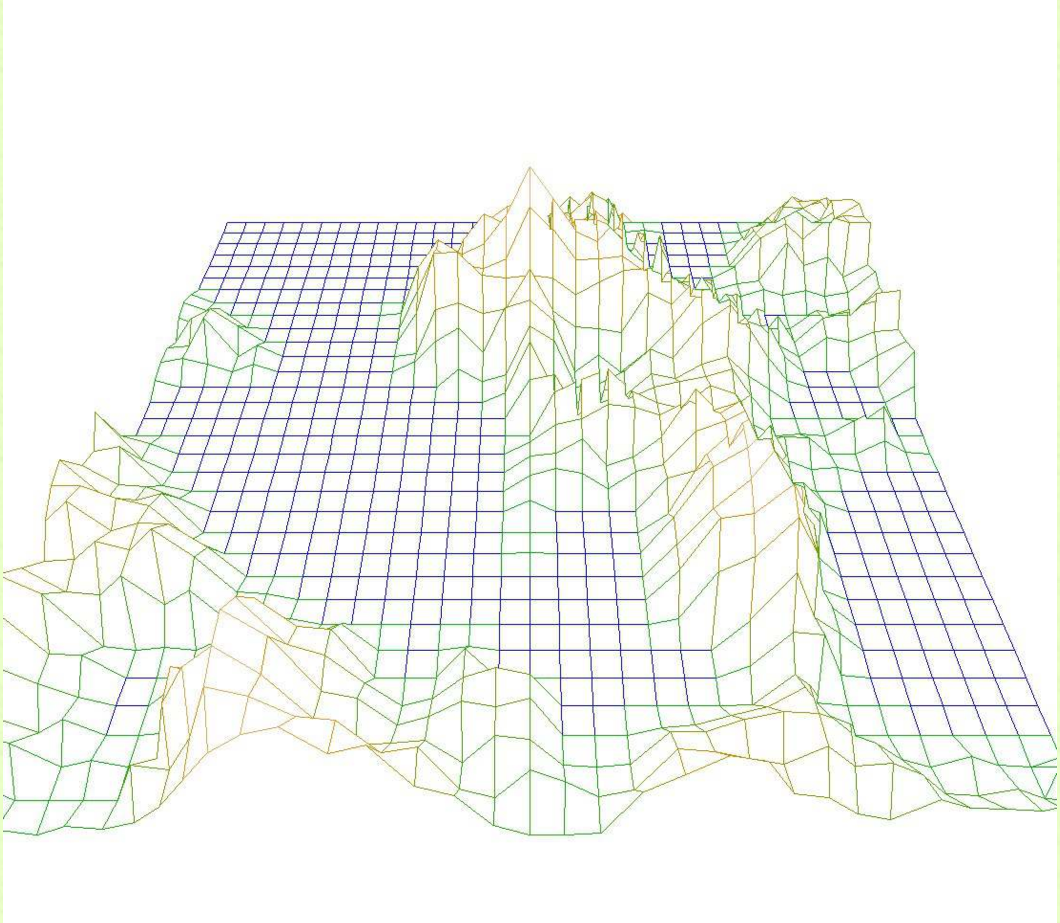
Landforms



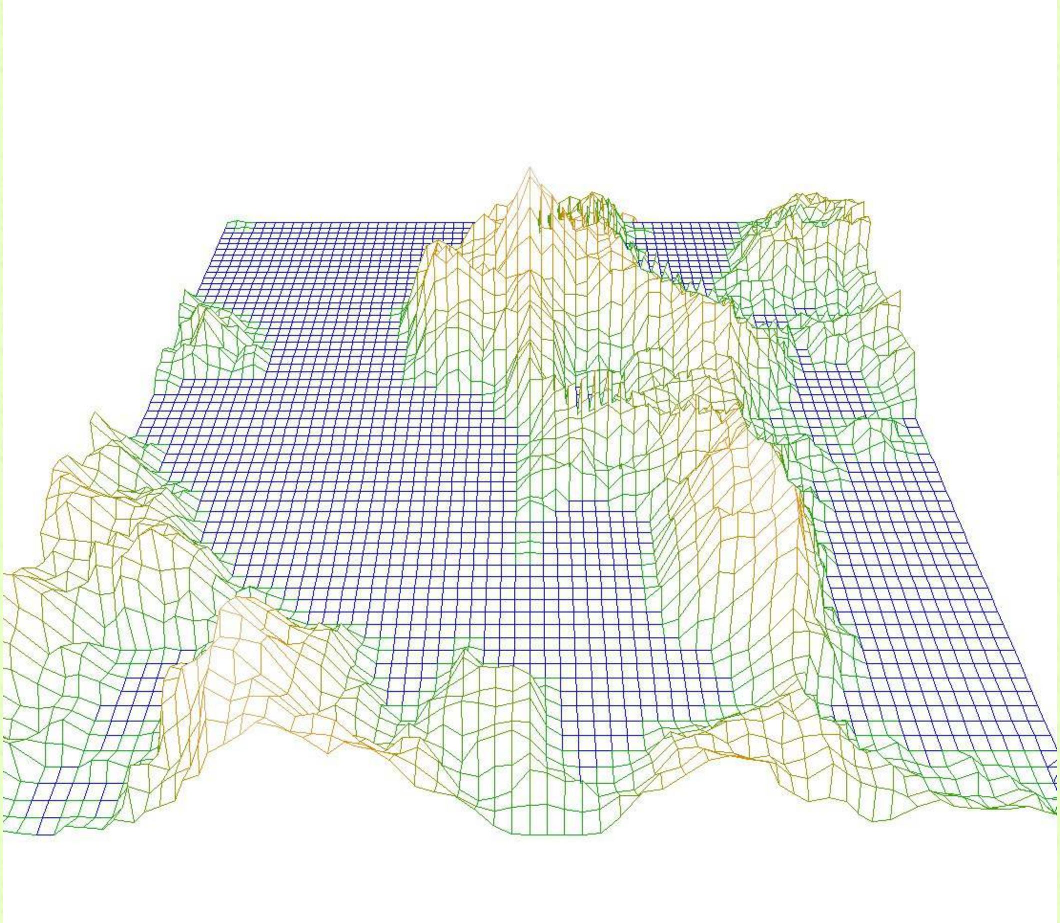
Landforms



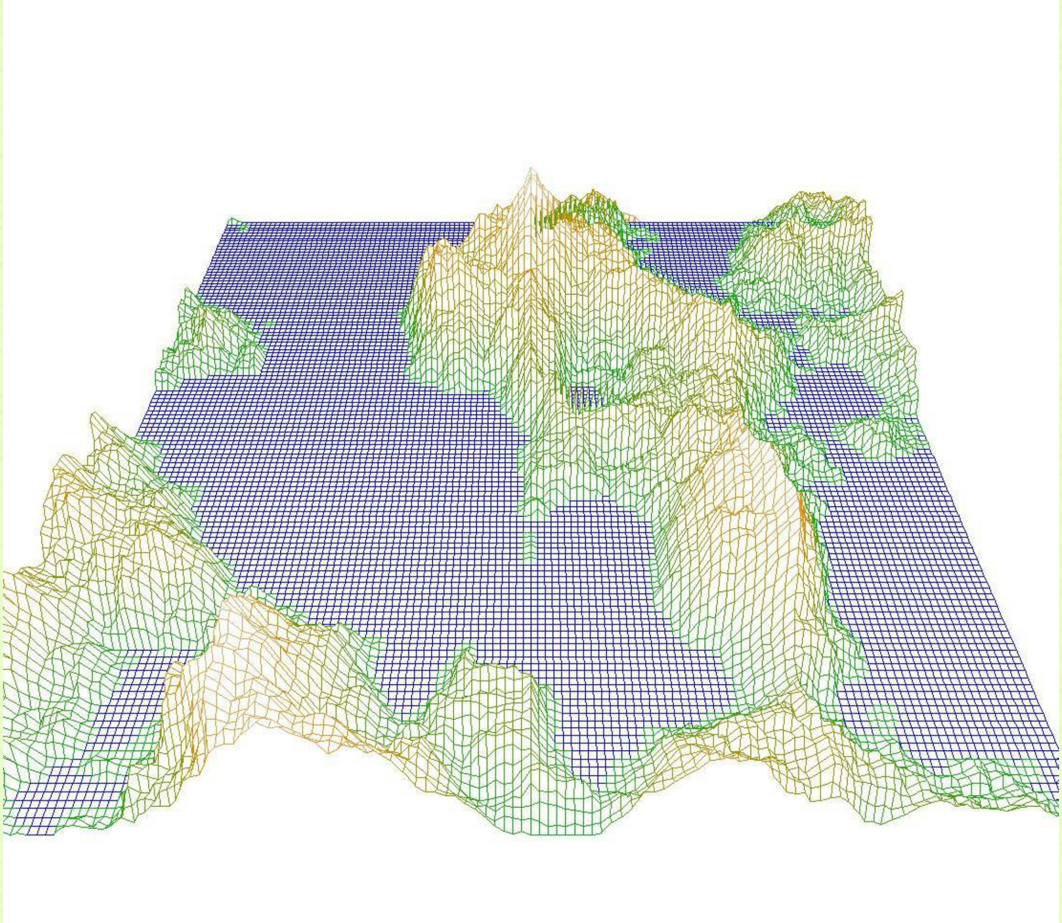
Landforms



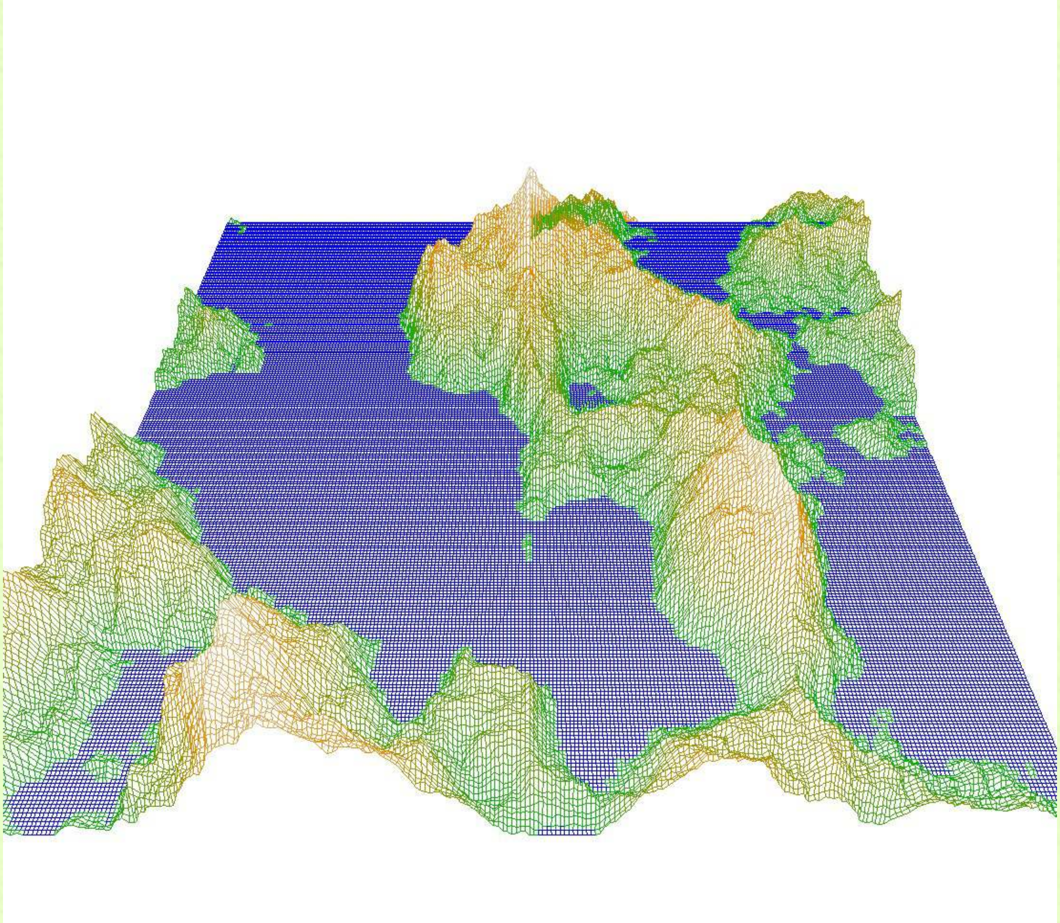
Landforms



Landforms



Landforms



Landforms

Adam Brown
2003

Landforms

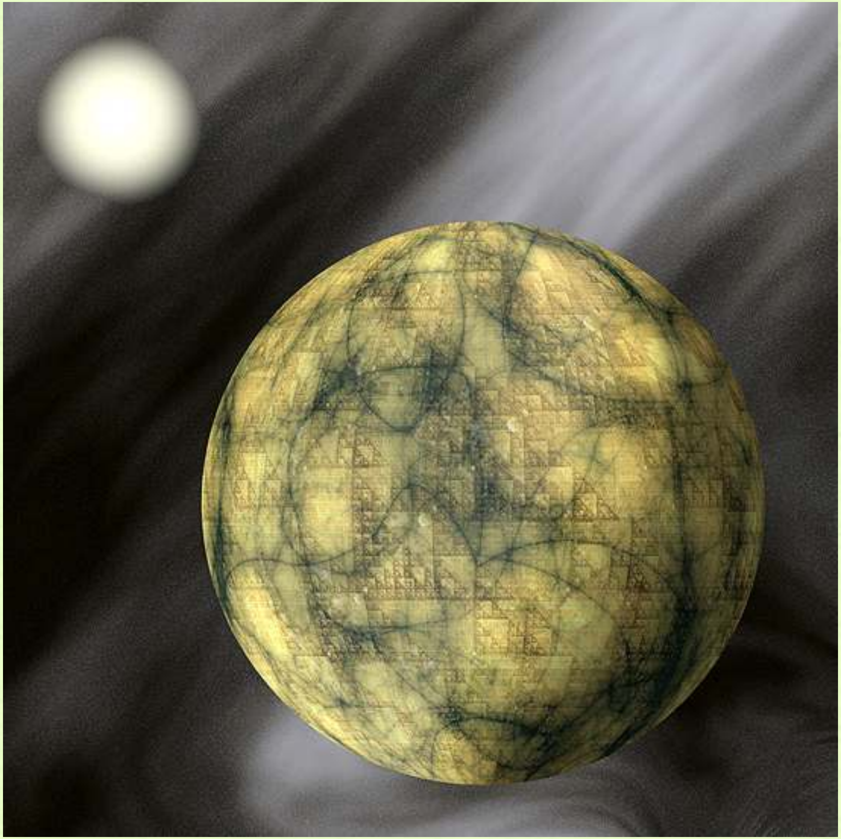
Landforms

Landforms

Landforms

Adam Brown
2002

Landforms



Sir Pinsky's Planet
2001 Daniel Kuzmenka

What is a fractal? A rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole.

Benoit B. Mandelbrot