Nature uses as little as possible of anything.
Johannes Kepler

An Empirical Beginning

Hi AST 111. In this module well go after “What are Kepler's three laws of planetary motion?”

In summary, Kepler's three laws — we’re going to walk through each of these in a little bit of detail and i just want to get all three out there to give a start-- are

Number one. The orbits of the planets are ellipses. Not circles, but ellipses, with the sun at one focus.

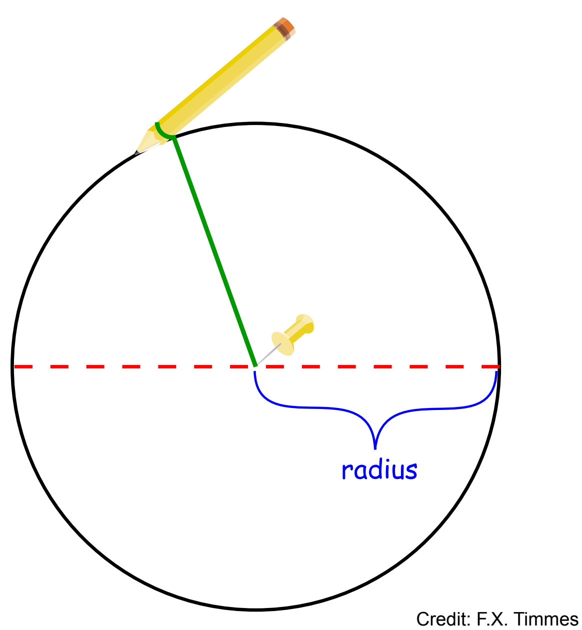
Number two. As a planet moves around its orbit, it sweeps out equal area in equal time.

Number three, the one with the meat on it, is that the more distant planets orbit the sun at slower speeds, obeying the relationship

$$P^2 = a^3$$

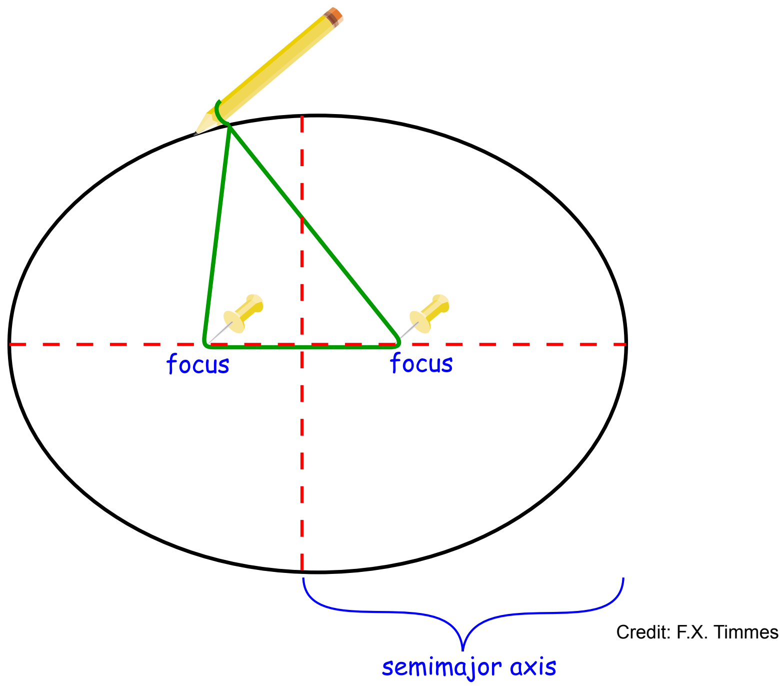
that he period squared is equal to the semi-major axis cubed. P is the time to complete an orbit, in years. It's on the left hand side of the equation. On the right hand side is semi major axis in astronomical units. And you know what an astronomical unit is; it's the average distance between the Earth and the Sun. And so this is a space - time relationship. On the left hand side is a time, and on the right hand side is a space coordinate. *P* squared is *a* cubed.

We’ll walk through each of Kepler’s laws in more detail.



Drawing a circle

You probably know how to make the circle. One way is to take a string and a pencil and tack the end of the string down, make the string tight, then carve out the shape that you get. And what you get is a circle. We call that distance from where you stick the pin it the mark made by the pencil the radius.



Drawing an ellipse

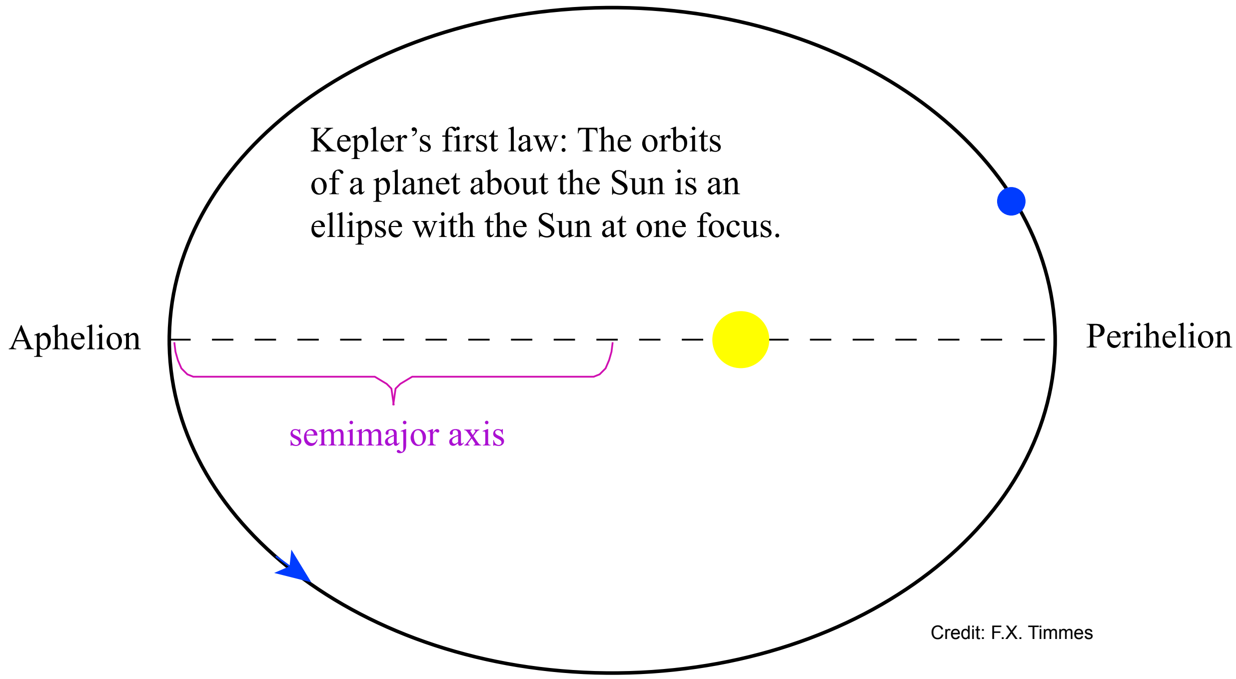
Now suppose you don't do one pin, but you do two pins. So you plop down two pins and you put the string around the pins. Keep the string tight, carve it out, and the object that you get is referred to as an ellipse.

So an ellipse is a generalization of the circle. If you like, it's got two centers, which we call the foci. And as you move the foci closer and closer together, of course, the ellipse becomes the circle.

If you take the foci and you spread them out really, really far, they get farther and farther apart, the ellipse gets skinnier and skinnier until, if you to take them out far enough, the ellipse begins to look like a line.

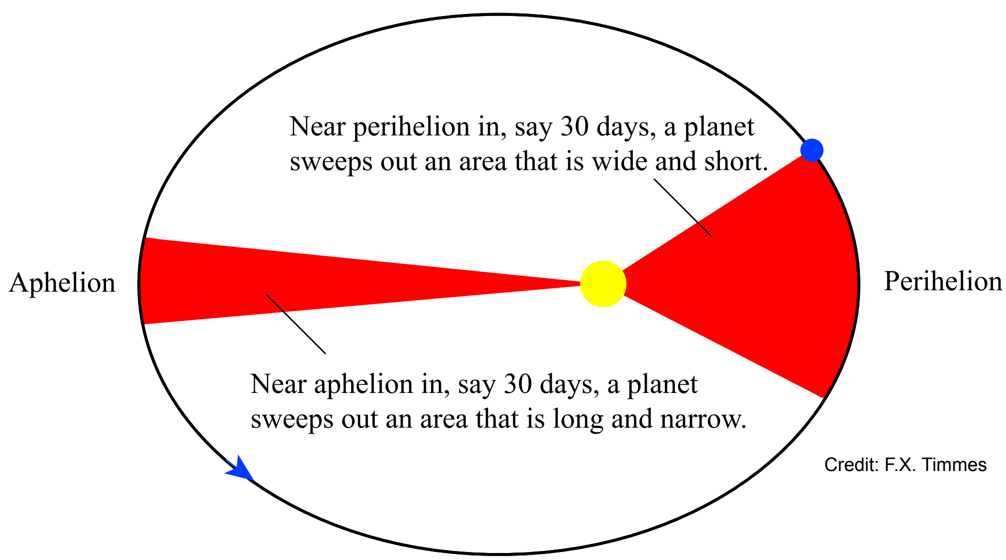
The distance from the center of the ellipse to the the long side of the ellipse is called the semimajor access. There's also a semiminor axis, which is the distance from the center out to the short, squat side of the ellipse. But we're not going to be concerned about that. We're mainly concerned about the semimajor axis.

So Kepler's first law is that the orbits of the planet about the Sun is an ellipse, with the sun at one focus. The image above shows the semimajor axis, and the sun at one of the foci. There's nothing at the other foci. That's perfectly fine. Nothing needs to be there.



The distance where the planet is closest to the sun is is called perihelion. The distance farthest away from the sun is called aphelion. The other important interpretation of the semimajor axis is that it is the average distance from the sun to a planet. So we have two definitions of the semimajor axis. Geometrically it is the distance from the center of the ellipse to the curve along the long axis of the ellipse. But it's also the average distance between the sun and the planet. Super convenient.

The reason why Kepler's first law is important philosophically is because it gets rid of the idea that the



The areas swept out in any 30 day period are equal.

heavenly bodies follow perfect circles. It's an ellipse, a generalization of a circle, but it's not a perfect circle.

Kepler's second law, as shown above says equal area in equal time. For example, there in the figure, the yellow dot is the Sun. Say the blue dot is Earth going around. Near perihelion, say you decide to wait 30 days. the 30 days is not special, it could be 2 minutes or whatever time you want. I'm just using 30 days as illustrative. As the Earth moves in around its orbit in 30 days, because it's near perihelion, it's going to move faster. In some sense, the gravity is stronger and so it's going to sweep out a short squat triangle.

Near aphelion, when Earth is farther away from the Sun, Earth will be moving slows because in some sense gravity is weaker. And the triangle you carve out in 30 days is a long skinny one. But the area between those two triangles - short squat and long skinny, are equal. Equal area in equal time.

This is important from a philosophical point of view because it abandons the ancient Greek belief that the planets have to move at constant uniform speed. Sometimes a planet goes afster, sometimes slower. So we are abandoning, finally, that idea of perfect uniform motion.

Finally Kepler's third law is *P* squared is equal to *a* cubed, where *P* is the orbital period, in years, and *a* is the semimajor axis in AU.

Now the left hand side, the period, that's easy to measure. Just break out your stopwatch. You say, OK, I'm going to watch Mars go around. Click Click. The time it takes to go around is easy to measure. The right hand side is something very hard to get a handle on. How far away is that sticky dot on the celestial sphere? That's a very hard question to answer in Astronomy. And so here you have something that's easy to measure and you can relate it directly to something that's very hard to measure.

Kepler found that Mars had a period, from Tycho's data, of 1.881 years and the question how far away is Mars from the sun. This is a typical kind of question that you'll see on the quantitative problems and homework in this class. It's essential algebra.

Always start off by writing down the equation that you're using. Don't start plugging in numbers until you write down the relationship that you're using. So *P* squared is *a* cubed.

$$\mathrm{P}^2 = \mathrm{a}^3$$

Now plug-in *P*=1.881 squared is equal to a cubed. Break out your calculator, punch that in, and you'll get that 3.538 is equal to *a* cubed. To get rid of the cube, take the cube-root of both sides.

$$1.881^2 = \mathrm{a}^3 \qquad \mathrm{a}^3 = 3.538$$

$$\mathrm{a} = 3.538^{1/3} = 1.524 \text{ AU}$$

So for the first time in human history, Kepler knew how far away Mars was from the Sun. It's about 1.5, times as far from the Sun as the Earth.

In summary, Kepler's three laws are 1) ellipses, 2) equal area equal times, and 3) *P* squared is *a* cubed. OK?

Thanks! Bye Bye.