Abundance Variables - 12Aug2017

Baryon number is an invariant. Define the abundance of species Y_i by

$$Y_i = \frac{n_i}{n_B} = \frac{N_i}{N_B} \tag{1}$$

where N_i is the number of particles of isotope i, N_B is the number of baryons, n_i is the number density [cm^{-3}] of isotope i and n_B is baryon number density [cm^{-3}]. The number of baryons in isotope i divided by the total number of baryons is the baryon fraction X_i,

$$X_i = Y_i\, A_i = \frac{n_i\, A_i}{n_B} \tag{2}$$

where A_i is the atomic mass number, the number of baryons in an isotope. Usually the baryon fraction is called the "mass fraction". Note

$$\sum X_i = \frac{n_B}{n_B} = 1 \tag{3}$$

is invariant under nuclear reactions. Define the baryon density, in atomic mass units, as

$$\rho_B = n_B\, m_u = \frac{n_B}{N_A} \quad \text{g cm}^{-3} \tag{4}$$

where m_u is the atomic mass unit [g] and N_A is the Avogadro number [g^{-1}] in a system of units where the atomic mass unit is *defined* as 1/12 mass of an unbound atom of ^{12}C is at rest and in its ground state.

Mean atomic number

$$\overline{\mathrm{A}} = \frac{\sum n_i \mathrm{A}_i}{\sum n_i} = \frac{n_B}{\sum n_i} = \frac{\sum Y_i \mathrm{A}_i}{\sum Y_i} = \frac{1}{\sum Y_i} \tag{5}$$

Mean charge

$$\overline{\mathrm{Z}} = \frac{\sum n_i \mathrm{Z}_i}{\sum n_i} = \frac{\sum Y_i \mathrm{Z}_i}{\sum Y_i} = \overline{\mathrm{A}} \sum Y_i \mathrm{Z}_i \tag{6}$$

Electron to baryon ratio, where the second equality assumes full ionization

$$Y_e = \frac{n_e}{n_B} = \frac{\sum n_i Z_i}{n_B} = \sum Y_i Z_i = \frac{\overline{\mathrm{Z}}}{\overline{\mathrm{A}}} \tag{7}$$

Neutron excess

$$\eta = \sum (\mathrm{N}_i - \mathrm{Z}_i) Y_i = \sum (\mathrm{A}_i - 2\mathrm{Z}_i) Y_i = \sum \mathrm{A}_i Y_i - 2Y_e = 1 - 2Y_e \tag{8}$$

Mean ion molecular weight

$$\mu_{\mathrm{ion}} = \overline{\mathrm{A}} \tag{9}$$

Mean electron molecular weight

$$\mu_{\mathrm{ele}} = \frac{1}{Y_e} = \frac{\overline{\mathrm{A}}}{\overline{\mathrm{Z}}} \tag{10}$$

Mean molecular weight

$$\mu = \left[\frac{1}{\mu_{ion}} + \frac{1}{\mu_{ele}}\right]^{-1} = \left[\frac{1}{\overline{\mathrm{A}}} + Y_e\right]^{-1} = \left[\frac{1}{\overline{\mathrm{A}}} + \frac{\overline{\mathrm{Z}}}{\overline{\mathrm{A}}}\right]^{-1} = \frac{\overline{\mathrm{A}}}{\overline{\mathrm{Z}} + 1} = \frac{n_B}{\sum n_i + n_e} \tag{11}$$