*RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.**

II.

Brightness and Spectral Class.

HAVING thus made a rapid survey of the general field, I shall now ask your attention in greater detail to certain relations which have been the more special objects of my study.

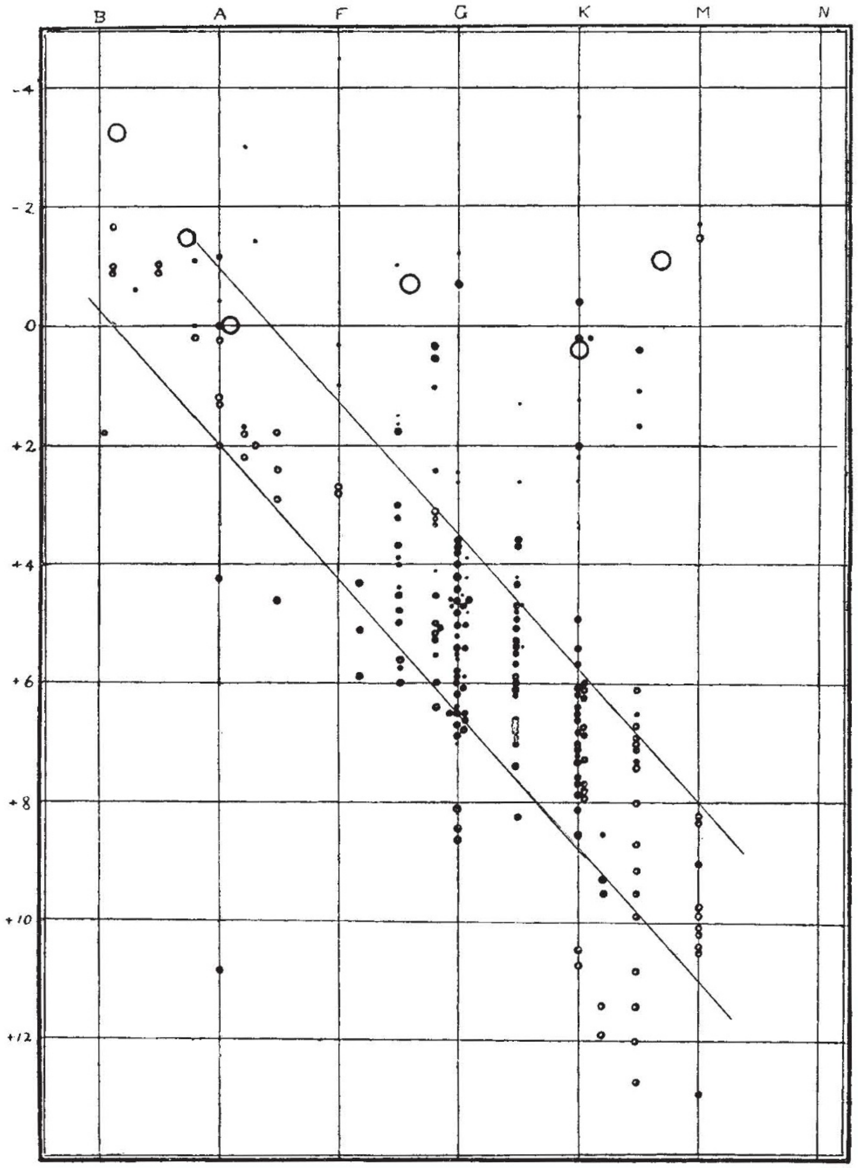

Fig. 1.

Let us begin with the relations between the spectra and the real brightness of the stars. These have been discussed by many investigators—notably by Kapteyn and Hertzsprung—and many of the facts which will be brought before you are not new; but the observational material here presented is, I believe, much more extensive than has hitherto been assembled. We can only determine the real brightness of a star when we know its distance; but the recent accumulation of direct measures of parallax, and the discovery of several moving clusters of stars the distances of which can be determined, put at our disposal far more extensive data than were available a few years ago.

Fig. 1 shows graphically the results derived from all the direct measures of parallax available in the spring of 1913 (when the diagram was constructed). The spectral class appears as the horizontal coordinate, while the vertical one is the absolute magnitude, according to Kapteyn's definition—that is, the visual magnitude which each star would appear to have if it should be brought up to a standard distance, corresponding to a parallax of 0·1″ (no account being taken of any possible absorption of light in space). The absolute magnitude, −5, at the top of the diagram, corresponds to a luminosity 7500 times that of the sun, the absolute magnitude of which is 4·7. The absolute magnitude 14, at the bottom, corresponds to 1/5000 of the sun's luminosity. The larger dots denote the stars for which the computed probable error of the parallax is less than 42 per cent. of the parallax itself, so that the probable error of the resulting absolute magnitude is less than ±1·0m. This is a fairly tolerant criterion for a "good parallax," and the small dots, representing the results derived from the poor parallaxes, should scarcely be used as a basis for any argument. The solid black dots represent stars the parallaxes of which depend on the mean of two or more determinations; the open circles, those observed but once. In the latter case, only the results of those observers whose work appears to be nearly free from systematic error have been included, and in all cases the observed parallaxes have been corrected for the probable mean parallax of the comparison stars to which they were referred. The large open circles in the upper part of the diagram represent mean results for numerous bright stars of small proper-motion (about 120 altogether) the observed parallaxes of which scarcely exceed their probable errors. In this case the best thing to do is to take means of the observed parallaxes and magnitudes for suitable groups of stars, and then calculate the absolute magnitudes of the typical stars thus defined. These will not exactly

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* An address delivered before a joint meeting of the Astronomical and Astrophysical Society of America and Section A of the American Association for the Advancement of Science, at Atlanta, Georgia, December 30, 1913, with a few additions, by Prof. H. N. Russell. Continued from p. 230.

correspond to the mean of the individual absolute magnitudes which we could obtain if we knew all the parallaxes exactly, but they are pretty certainly good enough for our purpose.

Upon studying Fig. 1 several things can be observed.

(1) All the white stars, of Classes B and A, are bright, far exceeding the sun; and all the very faint stars—for example, those less than 1/50 as bright as the sun—are red, and of Classes K and M. We may make this statement more specific by saying, as Hertzsprung does,[16] that there is a certain limit of brightness for each spectral class, below which stars of this class are very rare, if they occur at all. Our diagram shows that this limit varies by rather more than two magnitudes from class to class. The single apparent exception is the faint double companion to O_2 Eridani, concerning the parallax and brightness of which there can be no doubt, but the spectrum of which, though apparently of Class A, is rendered very difficult of observation by the proximity of its far brighter primary.

(2) On the other hand, there are many red stars of great brightness, such as Arcturus, Aldebaran, and Antares, and these are as bright, on the average, as the stars of Class A, though probably fainter than those of Class B. Direct measures of parallax are unsuited to furnish even an estimate of the upper limit of brightness to which these stars attain, but it is clear that some stars of all the principal classes must be very bright. The range of actual brightness among the stars of each spectral class therefore increases steadily with increasing redness.

(3) But it is further noteworthy that all the stars of Classes K5 and M which appear on our diagram are either very bright or very faint; there are none comparable with the sun in brightness. We must be very careful here not to be misled by the results of the methods of selection employed by observers of stellar parallax. They have for the most part observed either the stars which appear brightest to the naked eye, or stars of large proper-motion. In the first case, the method of selection gives an enormous preference to stars of great luminosity, and, in the second, to the nearest and most rapidly moving stars, without much regard to their actual brightness. It is not surprising, therefore, that the stars picked out in the first way (and represented by the large circles in Fig. 1) should be much brighter than those picked out by the second method (and represented by the smaller dots). But if we consider the lower half of the diagram alone, in which all the stars have been picked out for proper-motion, we find that there are no very faint stars of Class G, and no relatively bright ones of Class M. As these stars were selected for observation entirely without consideration of their spectra (most of which were then unknown) it seems clear that this difference at least is real, and that there is a real lack of red stars comparable in brightness with the sun, relatively to the number of those 100 times fainter.

The appearance of Fig. 1 therefore suggests the hypothesis that, if we could put on it some thousands of stars instead of the 300 now available, and plot their absolute magnitudes without uncertainty arising from observational error, we would find the points representing them clustered principally close to two lines, one descending sharply along the diagonal, from B to M, the other starting also at B, but running almost horizontally. The individual points, though thickest near the diagonal lines, would scatter above and below it to a vertical distance corresponding to at least two magnitudes, and similarly would be thickest near the horizontal line, but scatter above and below it to a distance which cannot so far be definitely specified, so that there would be two fairly broad bands in which most of the points lay. For Classes A and F these two zones would overlap, while their outliers would still intermingle in Class G, and probably even in Class K. There would, however, be left a triangular space between the two zones, at the right-hand edge of the diagram, where very few (if any) points appeared, and the lower left-hand corner would be still more nearly vacant.

We may express this hypothesis in another form by saying that there are two great classes of stars, one of great brightness (averaging, perhaps, a hundred times as bright as the sun), and varying very little in brightness from one class of spectrum to another; the other of smaller brightness, which falls off very rapidly with increasing redness. These two classes of stars were first noticed by Hertzsprung,[17] who has applied to them the excellent names of *giant* and *dwarf* stars. The two groups, on account of the considerable internal differences in each, are only distinctly separated among the stars of Class K or redder. In Class F they are partially, and in Class A thoroughly, intermingled, while the stars of Class B may be regarded equally well as belonging to either series.

In addition to the stars of directly measured parallax, represented in Fig. 1, we know with high accuracy the distances and real brightness of about 150 stars which are members of the four moving clusters the convergent points of which are known, namely, the Hyades, the Ursa Major group, the 61 Cygni group, and the large group in Scorpius, discovered independently by Kapteyn, Eddington, and Benjamin Boss, the motion of which appears to be almost entirely parallactic. The data for the stars of these four groups are plotted in Fig. 2, on the same system as in Fig. 1. The solid black dots denote the members of the Hyades; the open circles, those of the group in Scorpius; the crosses, the Ursa Major group; and the triangles, the 61 Cygni group. Our lists of the members of each group are probably very nearly complete down to a certain limiting (visual) magnitude, but fail at this point, owing to lack of knowledge regarding the proper motions of the fainter stars. The apparently abrupt termination of the Hyades near the absolute magnitude 7·0, and of the Scorpius group at 1·5, arises from this observational limitation.

The large circles and crosses in the upper part of Fig. 2 represent the absolute magnitudes calculated from the mean parallaxes and magnitudes of the groups of stars investigated by Kapteyn, Campbell, and Boss, concerning which data were given in Table III. The larger circles represent Boss's results, the smaller circles Kapteyn's, and the large crosses Campbell's.

It is evident that the conclusions previously drawn from Fig. 1 are completely corroborated by these new and independent data. Most of the members of these clusters are dwarf stars, and it deserves particular notice that the stars of different clusters, which are presumably of different origin, are similar in absolute magnitude. But there are also a few giant stars, epecially of Class K (among which are the well-known bright stars of this type in the Hyades); and most remarkable of all is Antares, which, though of Class M, shares the proper motion and radial velocity of the adjacent stars of Class B, and is the brightest star in the group, giving out about two thousand times the light of the sun.

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[16] *A. N.*, 4422, 1910.

[17] *Zeitschrift für Wissenschaftliche Photographie*, vol. iii., p. 442, 1905.

It is also clear that the naked-eye stars, studied by Boss, Campbell, and Kapteyn, are, for the most part, giants. With this in mind, we are now in a position to explain more fully the differences between the results of these investigators.

All the stars of Class B are giants, and, so far as we may judge from the Scorpius cluster, they do not differ from one another very greatly in absolute brightness. It is therefore natural that the results of all three investigators are in this case fairly similar, though Campbell, in employing stars that averaged brighter to the eye than did the others, has evidently been working with stars that are really brighter. In Class A the giants and dwarfs differ so little, and are so thoroughly intermingled, that the situation is about the same. In Class M, even the nearest and brightest of the dwarf stars are invisible to the naked eye: hence the stars of this class studied by the three investigators are all giants, and once more their results agree.

A number of the dwarf stars of Class K are visible to the naked eye; but these all lie very near us, and have such large proper motions that they are excluded as "abnormal" by both Campbell and Boss. The results of the two agree in indicating that the stars studied by them are typical giants. The few dwarfs, however, have such large parallaxes and proper-motions that their inclusion more than doubles the mean proper-motion, and presumably, also, the mean parallax of the whole, as shown by Kapteyn's figures in Table III. For Class G, the dwarf stars average much brighter, and a much greater number of them is visible to the naked eye. These have large parallaxes and proper-motions, and raise the average for all the stars of this class to greater values than for any other. But Boss's rigorous limitation to small proper-motions weeds them practically all out leaving giant stars once more. Campbell's less drastic procedure omits only the nearer of the dwarfs (to be precise, those with the larger proper-motions), and his result lies about half-way between the others. In the case of Class F, the dwarf stars are still brighter—intermingling, in fact, with the giants. We can therefore see them farther off, and we get more of them in our catalogues, in proportion to the giants, than in any other class. Their mean parallax is, however, smaller than for the dwarfs of Classes G and K, and hence the mean proper-motion and parallax of all the stars of this class is less than for Class G. Campbell's criterion here excludes very few stars, and even Boss's admits a good many of the remoter and slower moving dwarfs, causing his mean parallax and proper-motion to be considerably greater for this class than for any other.

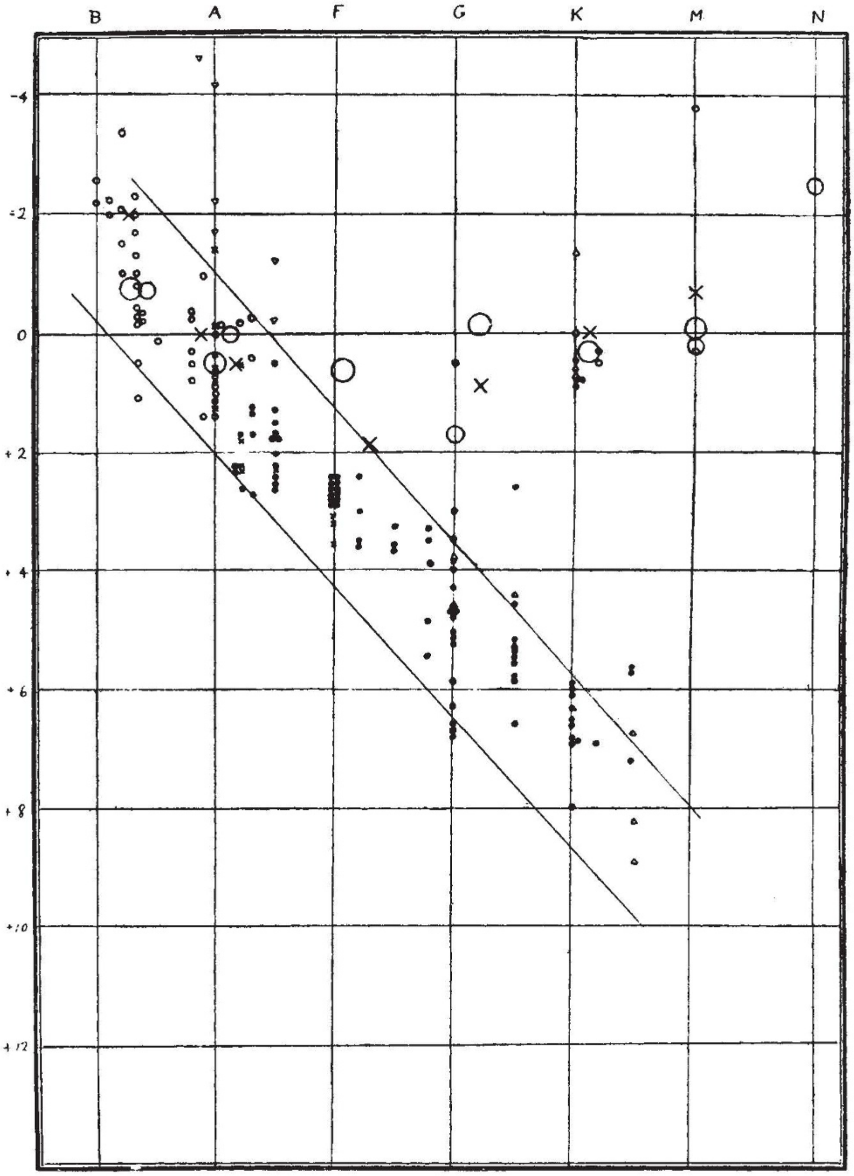

FIG. 2.

It should finally be added that Kapteyn's discussion shows that the stars of Class N are exceedingly bright, possibly surpassing any of the other giant stars.

We are now in a position to define more precisely the brightness of a typical giant or dwarf star of a given class of spectrum, and also to obtain a measure of the degree of divergence of the individual stars from this typical brightness. Taking first the stars of Class B and the dwarf stars of the other classes, we find, for the mean absolute magnitudes of all the stars of each class, the following values:—

Table V.
Mean Absolute Magnitudes.

Spectrum	Stars of measured parallax				Stars in clusters			
	No.	Abs. mag.	Formula	O–C m.	No.	Abs. mag.	Formula	O–C m.
B2	—	—	—	—	21	−1·2	−1·1	−0·1
B8	—	—	—	—	8	+0·3	+0·2	+0·1
A0	6	+1·4	+1.4	0·0	13	0·5	0·6	−0·1
A4	7	2·5	2·3	+0·2	26	1·7	1·5	+0·2
F0	—	—	—	—	15	2·4	2·7	−0·3
F1	5	4·2	3·7	+0·5	—	—	—	—
F3	—	—	—	—	7	3·3	3·3	0·0
F5	9	4·3	4·5	−0·2	—	—	—	—
F8	8	5·1	5·2	−0·1	5	4·2	4·4	−0·2
G0	29	5·7	5·6	+0·1	18	5·0	4·8	+0·2
G5	19	5·7	6·6	−0·9	9	5·1	5·8	−0·7
K0	28	7·1	7·7	−0·6	9	6·4	6·9	−0·5
K4	19	9·2	8·6	+0·6	7	+7·0	+7·7	(−0·7)
Ma	10	+9·9	+9·8	+0·1	—	—	—	—

The rate of decrease of brightness with increasing redness is very nearly the same for the stars with directly measured parallaxes and the stars in clusters, but the latter appear, with remarkable consistency, to be about 0·8m. brighter than the former. This seems at first sight very puzzling, but it is undoubtedly due to the way in which the stars observed for parallax were selected. Most observers, in preparing their working lists, have included mainly those stars which were brighter than a given magnitude and had proper-motions exceeding some definite limit. Of the stars above this limiting magnitude, those of greater actual luminosity will be, on the average, farther away, and have smaller proper-motions, than those of small luminosity, and selection by proper-motion favours the latter. The limitation of our present lists to stars the parallaxes of which have been determined with a probable error not exceeding 42 per cent. of their own amounts, though necessary to diminish the effects of casual errors of observation, works in the same direction, for, among the stars of any given visual magnitude, those of greatest luminosity have the smallest parallaxes, and are least likely to pass the test. The difference shown in our table need not therefore alarm us, but it is clear that the stars in clusters, rather than the others, should be taken as typical of the dwarf stars as a whole. For both sets of stars the absolute magnitude appears to be very nearly a linear function of the spectral class (if B is regarded as 1, A as 2, etc.) The columns headed "formula" in Table V. give the values calculated from the expressions M=1·4m.+2·1m. (Sp.−A) for the stars of directly measured parallax, and M=0·6m.+2·1m. (Sp.−A) for the stars in clusters. The residuals from these empirical formulæ, for the mean absolute magnitudes of the observed stars of different classes, average ±0·33m. in the first case and ±0·29m. in the second. They appear to be accidental in character, though in some cases (notably in Class G5) the residuals for the stars of the two sets are similar in sign and magnitude. The large negative residuals for Classes K and K5 in the clusters arise from the fact that in the Hyades, which contribute most of these stars, only the brighter ones have had their proper-motions determined, and get into our lists, as is clear from examination of Fig. 2.

Among the dwarf stars, therefore, a typical star of any spectral class is about seven times fainter than one of the preceding class, and seven times brighter than one of the following class.

The giant stars of all the spectral classes appear to be of about the same mean brightness, averaging a little above absolute magnitude zero, that is, about a hundred times as bright as the sun. Since the stars of this series which appear in Fig. 2 have been selected by apparent brightness, which gives a strong preference to those of the greatest luminosity, the average brightness of all the giant stars in a given region of space must be less than this, perhaps considerably so.

By tabulating the residual differences between the absolute magnitudes of the individual dwarf stars and the values given by the formulæ just described, we find that the average difference, regardless of sign, for the stars of measured parallax is ±0·88m. for spectra A to F8, ±1·02m. for spectra G and G5, and ±1·15m. for K and M. For the stars in clusters, the average differences are ±0·70m. for spectra BO to B9, ±0·66m. for A and A5, ±0·56m. for spectra F to F8, and ±0·80m. for G and G5.

These differences are larger for the stars of measured parallax than for the others (probably on account of the greater average uncertainty of the individual parallaxes and spectra in this case), but show no marked systematic variation with the class of spectrum. Their distribution follows very approximately the law of accidental errors, as is shown by Table VI., in which the observed numbers lying between certain limits are compared with those given by this law

Table VI.
Distribution of Differences from the Typical Absolute Magnitudes.

Stars with measured parallax			Stars in clusters		
Limits	Observed	Theory	Limits	Observed	Theory
m. m.			m. m.		
±0·0 to ±0·8	65	61	±0·0 to ±0·5	59	58
±0·8 to ±1·6	41	44	±0·5 to ±1·0	42	42
±1·6 to ±2·4	21	23	±1·0 to ±1·5	21	24
±2·4 to ±3·2	10	9	±1·5 to ±2·0	10	8
±3·2 to ±4·0	3	3	±2·0 to ±2·5	4	4

The theoretical distribution for the stars in clusters corresponds to a probable error of ±0·61m., and that for the others to one of ±0·94m. Correction for the known influence of uncertainties of the parallaxes and spectra would reduce the latter to about ±0·75m. It appears, therefore, that the absolute magnitude of a dwarf star can be predicted with surprising accuracy from a mere knowledge of its spectrum. Half of all the dwarf stars are not more than twice as bright or as faint as the typical stars of their spectral classes. The corresponding uncertainty in the estimated parallax would be about one-third of its amount.

The parallaxes of the giant stars are so small, in comparison with the errors of even the best present methods of observation, that direct observations are not well adapted to determine to what degree they differ in brightness among themselves. An indirect method of determining this is, however, practicable, among those classes in which all the naked-eye stars are giants, by comparing the parallactic motions of those stars the proper-motions of which at right angles to the direction of the parallactic drift are large and small. A discussion by this method of the typical case of Class M (the details of which will be given elsewhere) shows that, if the distribution of the absolute magnitudes of these stars also follows the "law of errors," the probable error corresponding to it is approximately ±0·6m.—almost exactly the same as has already been found for the dwarf stars. The mean absolute magnitude of all the stars of this class which are visible to the naked eye is −0·5, and that of all the stars in a given region of space is +0·6. This method can scarcely be applied to the naked-eye stars of the other spectral classes (unless some way can be devised for weeding out the dwarf stars from among the giants); but it seems probable that they do not differ greatly from the stars of Classes B and M as regards the degree of their similarity to one another in brightness With such a probable error of distribution of the absolute magnitudes as has here been derived, the giant and dwarf stars would overlap perceptibly in Class G, be just separated in Class K, and widely so in Class M, as the observational data indicate.

The questions now arise: What differences in their nature or constitution give rise to the differences in brightness between the giant and dwarf stars? and Why should these differences show such a systematic increase with increasing redness or "advancing" spectral type?

We must evidently attack the first of these questions before the second. The absolute magnitude (or the actual luminosity) of a star may be expressed as a function of three physically independent quantities—its mass, its density, and its surface-brightness. Great mass, small density, and high surface-brightness make for high luminosity, and the giant stars must possess at least one of these characteristics in a marked degree, while the dwarf stars must show one or more of the opposite attributes.

A good deal of information is available concerning all these characteristics of the stars. The masses of a considerable number of visual and spectroscopic binaries are known with tolerable accuracy, the densities of a larger number of eclipsing variable stars have recently been worked out, and the recent investigations on stellar temperatures lead directly to estimates of the relative surface brightness of the different spectral classes (subject, of course, to the uncertainty whether the stars really radiate like black bodies, as they are assumed to do). We will take these matters up in order.

First, as regards the masses of the stars, we are confined to the study of binary systems, which may or may not be similar in mass to the other stars. There appears, however, to be no present evidence at all that they are different from the other stars, and in what follows we will assume them to be typical of the stars as a whole.

The most conspicuous thing about those stellar masses which have been determined with any approach to accuracy is their remarkable similarity. While the range in the known luminosities of the stars exceeds a millionfold, and that in the well-determined densities is nearly as great, the range in the masses so far investigated is only about fiftyfold. The greatest known masses are those of the components of the spectroscopic binary and eclipsing variable V Puppis, which equal nineteen times that of the sun; the smallest masses concerning which we have any trustworthy knowledge belong to the faint components of ζ Herculis and Procyon, and are from one-third to one-fourth of the sun's mass. These are exceptional values, and the components of most binary systems are more nearly similar to the sun in mass.

There appears, from the rather scanty evidence at present available, to be some correlation between mass and luminosity. Those stars which are known to be of small mass (say, less than half the sun's) are all considerably fainter than the sun. On the other hand, Ludendorff[18] has shown conclusively that the average mass of the spectroscopic binaries of spectrum B (which are all of very great luminosity) is three times as great as that of the spectroscopic binaries of other spectral types, and may exceed ten times that of the sun. Further evidence in favour of this view is found in the fact that the components of a binary, when equal in brightness, are nearly equal in mass, while in unequal pairs the brighter star is almost (if not quite) always the more massive, but the ratio of the masses very rarely exceeds 3 : 1, even when one component is hundreds of times as bright as the other. Very large masses (such as one hundred times the sun's mass) do not appear, though they would certainly be detected among the spectroscopic binaries if they existed. It is equally remarkable that there is no trustworthy evidence that any visible star has a mass as small as one-tenth that of the sun. The apparent exceptions which may be found in the literature of the subject may be shown to arise from faulty determinations of parallax, arbitrary estimates of quantities unobtainable by observation (such as the ratio of the densities of the two components of Algol), and even numerical mistakes.

It follows from this similarity of mass that we can obtain a very fair estimate of the parallax of any visual binary (called by Doberck the hypothetical parallax) by guessing at its mass, and reversing the familiar relation between mass and parallax. If we assume that the mass of the system is twice that of the sun (about the average value), our hypothetical parallaxes, as the existing evidence shows, will usually be well within 40 per cent. of the truth, and the deduced absolute magnitudes of the components will rarely be more than one magnitude in error. We may thus extend our study of the relation between absolute magnitude and spectrum to all the visual binaries for which orbits have been computed. The hypothetical absolute magnitudes which we will obtain for them will indeed be somewhat in error, owing to the differences in their masses; but, for our present purpose, *the hypothetical values are actually more useful than the true values would be*. This sounds remarkable; but it is easy to show that, if we assume that the brighter components of the systems have all the same mass (say that of the sun), the resulting hypothetical absolute magnitudes will be the actual absolute magnitudes of stars identical in density and surface-brightness with the real stars, but all of the assumed mass. In other words, *the effects of differences of mass among the stars are eliminated from these hypothetical absolute magnitudes*, leaving only those of differences in density and surface-brightness. (This is simply a statement in different form of a theorem which has been known for many years.) It is therefore desirable to extend our study to as many binary stars as possible. The number for which binary orbits have been computed is relatively small, but by a simple statistical process we may include all those pairs which are known to be connected really physically, however slow their relative motion may be.[19]

Consider any pair of stars, of combined mass m times that of the sun, at a distance of r astronomical units, and with a relative velocity of v astronomical units per annum. By gravitational theory, we have

$$v^2r=(2\pi)^2m(2-r/a)=39{\cdot}7m(2-r/a),$$

where a is the semi-major axis of the orbit Now let π be the parallax of the system, s the observed distance in seconds of arc, w the observed relative motion in seconds of arc per annum, and i_1 and i_2 the angles which r and v make with the line of sight. Then $s=r\pi\sin i_1$, $w=v\pi\sin i_2$, and our equation becomes

$$sw^2=39{\cdot}7\pi^3m\sin i_1\sin^2 i_2(2-r/a).$$

In the individual case, the last three factors of the second member are unknown, and we are no wiser

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[18] *A. N.*, 4520, 1911.

[19] An outline of this method was given by the speaker at the meeting of the Astronomical and Astrophysical Society of America at Ottawa, August 25, 1911, and published in *Science*, N.S., vol. xxxiv., pp. 523–25, October 20, 1911. A similar method was worked out quite independently and almost simultaneously by Hertzsprung, and published in *A. N.*, December 19, 1911 (the date of writing being October 11, 1911).

than at the start; but the average value which their product should have, in a large number of cases, and the percentage of these cases in which it should lie within any given limits, may be computed on the principles of geometrical probability. It is thus found that the formula $\pi^3 = sw^2/14{\cdot}6m$ gives values for the hypothetical parallax the average for a large number of cases of which will be correct, and that, while in individual cases these values will be too large or too small, half of them will be within 19 per cent. of the true values, and the numbers of larger errors will fall off in very nearly the manner corresponding to this probable error. If we compute absolute magnitudes from these parallaxes, their average for all the stars will be a little too bright (since the cases in which the computed parallax comes out too small have more influence than those in which it is too large). This may be allowed for by adding 0·15m. to all the hypothetical magnitudes so computed—an amount almost negligibly small for our present purpose.

We thus obtain a series of hypothetical absolute magnitudes the average for a large number of cases of which will be correct. In 59 per cent. of the individual cases the error arising from the statistical process—that is, from the substitution of a mean value of

$$\sin i_1 \sin^2 i_2 (2 - r/a)$$

for the true value—will affect the deduced magnitude by less than ±0·5m., and in 89 per cent. of all cases the error will not exceed ±1·0m. The approximation is therefore quite sufficient for our purpose. It should, however, be noted that, while the error of the statistical process can never make the computed absolute magnitude of any star too faint by more than 1·5m., it may in rare cases make it too bright by any amount whatever—more than 2·0m. in one case in sixty, more than 3·0m. once in 250 cases, and so on.

We may now proceed to compute hypothetical absolute magnitudes for all the physical pairs which show even a trace of relative motion—including many which are ordinarily described as "fixed," but, on careful study of the observations, show very slow relative change. With the aid of the splendid collection of observational data contained in Burnham's great catalogue and other recent works on double stars, and of many observations of spectra made at Harvard in generous response to requests for information, it has been possible to derive results for more than 550 stars. Assuming that the brighter component of each of these (which is usually the only one of which the spectrum is known) is equal in mass to the sun, estimating that of the fainter component on the basis of the difference of brightness (with the data for the systems in which the mass-ratio is known as a sufficient guide), and proceeding as indicated above, we obtain the data plotted in Fig. 3. The co-ordinates have here the same meaning as in the previous diagrams, and the figure shows at a glance the relations which would exist between the absolute magnitudes and spectra of these 550 stars if all differences of mass were eliminated, leaving only those of density and surface-brightness operative. Binaries for which orbits have been computed are shown by solid dots, and physical pairs, to which the statistical process has been applied, by open circles.

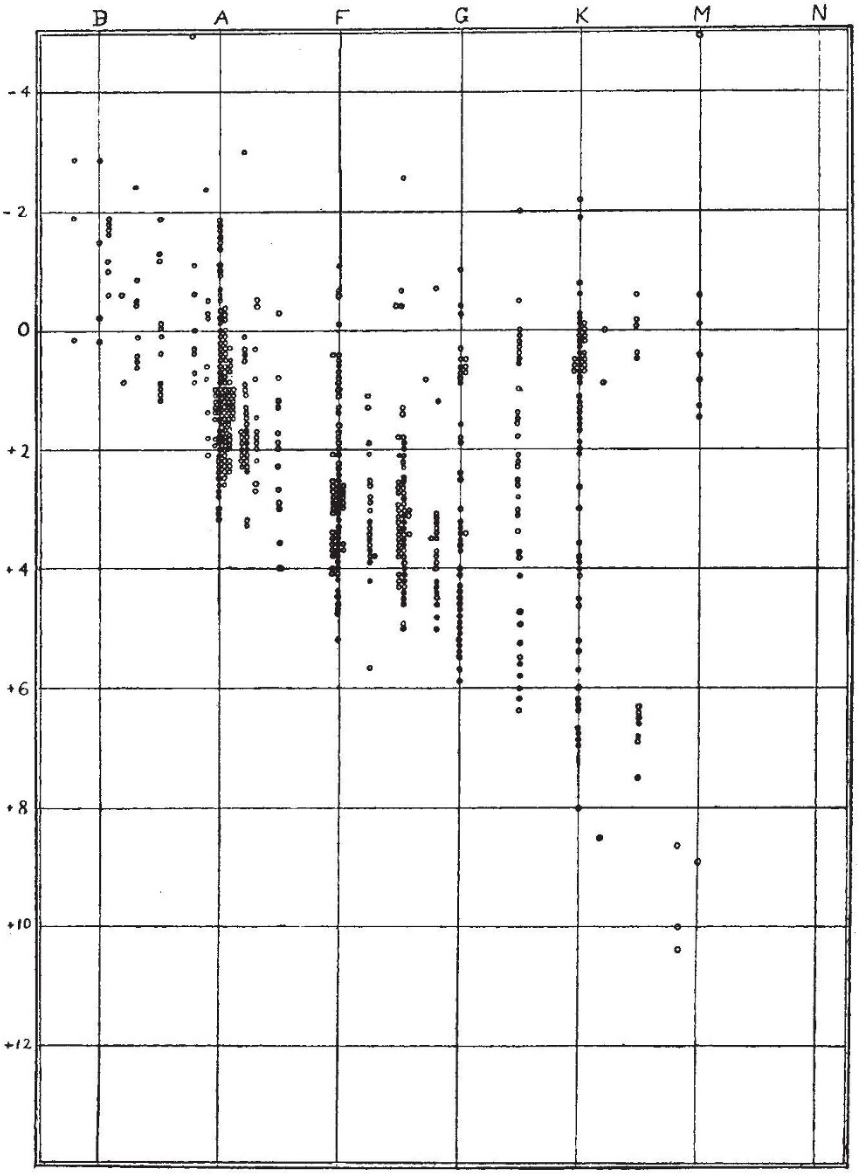

FIG. 3

Our new diagram is strikingly similar in appearance to the previous ones, even in its minor details.

The two series of giant and dwarf stars appear once more; the giants are all of about the same brightness, except that those of Class B are brighter than the rest; the dwarf stars diminish in brightness by about two magnitudes for each spectral class; the two series overlap up to Class G and separate at Class K, and so on. We have clearly come, for the third time, and again from independent data, upon the same phenomena as before; and, with the more extensive observational material, some of the characteristics and relations of the two groups are shown better than ever.

But this new evidence does much more than to confirm that which we have previously considered—it proves that the distinction between the giant and dwarf stars, and the relations between their brightness and spectral types, do not arise (primarily at least) from differences in mass. Even when reduced to equal masses, the giant stars of Class K are about one hundred times as bright as the dwarf stars of similar spectrum, and for Class M the corresponding ratio is fully 1000. Stars belonging to the two series must therefore differ greatly either in surface brightness or in density, if not in both.

There is good physical reason for believing that stars of similar spectrum and colour-index are at least approximately similar in surface brightness, and that the surface brightness falls off rapidly with increasing redness. Indeed, if the stars radiate like black bodies, the relative surface brightness of any two stars should be obtainable by multiplying their relative colour-index by a constant (which is the ratio of the mean effective photographic wave-length to the difference of the mean effective visual and photographic wave-lengths, and lies usually between 3 and 4, its exact value depending upon the systems of visual and photographic magnitude adopted as standards). Such a variation of surface brightness with redness will evidently explain at least the greater part of the change in absolute magnitude among the dwarf stars (as Hertzsprung and others have pointed out), but it makes the problem of the giant stars seem at first sight all the more puzzling.

The solution is, however, very simple. If a giant star of Class K, for example, is one hundred times as bright as a dwarf star of the same mass and spectrum, and is equal to it in surface brightness, it must be of ten times the diameter and 1/1000 of the density of the dwarf star. If, as in Class M, the giant star is one thousand times as bright as the dwarf, it must be less than 1/30,000 as dense as the latter. Among the giant stars in general, the diminishing surface brightness of the redder stars must be compensated for by increasing diameter, and therefore by rapidly decreasing density (since all the stars considered have been reduced to equal mass).

But all this rests on an assumption which, though physically very probable, cannot yet be said to be proved; and its consequences play havoc with certain generally accepted ideas. We will surely be asked, Is the assumption of the existence of stars of such low density a reasonable or probable one? Is there any other evidence that the density of a star of Class G or K may be much less than that of the stars of Classes B and A? Can any other evidence than that derived from the laws of radiation be produced in favour of the rapid decrease of surface brightness with increasing redness?

We can give at once one piece of evidence bearing on the last question. The twelve dwarf stars of Classes K2 to M, shown in Fig. 3, have, when reduced to the sun's mass, a mean absolute magnitude of 7·8—three magnitudes fainter than the sun. If of the sun's surface brightness, they would have to be, on the average, of one-fourth its radius, and their mean density would be sixty-four times that of the sun, or ninety times that of water—which is altogether incredible. A body of the sun's mass and surface brightness, even if as dense as platinum, would only be two magnitudes fainter than the sun, and the excess of faintness of these stars beyond this limit can only be reasonably ascribed to deficiency in surface brightness. For the four stars of spectra K8 and M, the mean absolute magnitude of which, reduced to the sun's mass, is 9·5, the mean surface brightness can at most be one-tenth that of the sun.

(*To be continued.*)

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

CAMBRIDGE.—The office of superintendent of the museum of zoology will shortly become vacant by the resignation of Dr. Doncaster. The stipend at present attached to the office is 200*l.* per annum.

Applications to occupy the University's table in the Zoological Station at Naples, and that in the laboratory of the Marine Biological Association at Plymouth should be addressed to Prof. Langley, The Museums, Cambridge, on or before June 4.

Mr. C. G. Darwin, eldest son of the late Sir George Darwin, has been appointed mathematical lecturer at Christ's College.

GLASGOW.—It is announced that honorary degrees are to be conferred on Dr. Archibald Barr, late regius professor of civil engineering and mechanics in the University, Colonel Sir William B. Leishman, F.R.S., professor of pathology in the Royal Army Medical College, and Sir Ernest H. Shackleton, C.V.O. The degrees will be conferred on Commemoration Day, June 23, when an oration on Lord Lister will be delivered by Sir Hector C. Cameron.

LONDON.—The Page-May Memorial Lectures for the current session will be delivered by Dr. Keith Lucas, whose subject will be "The Conduction of the Nervous Impulse." The course will be held at University College, on Fridays, beginning on May 15. The lectures are open to all internal students of the University of London and to such other persons as are specially admitted. Applications should be addressed to the secretary, University College, London (Gower Street, W.C.).

OXFORD.—Congregation on May 5 passed a statute authorising the establishment of an additional professorship in chemistry, to be called Dr. Lee's Professorship of Chemistry. In the same Congregation the statutes providing for the establishment of Dr. Lee's Professorships of Anatomy and Experimental Philosophy, in place of the existing Lee's Readerships, passed their first stage. Should these statutes be finally approved, the University will be relieved of its present contribution of 1470*l.* towards the stipends of the professors of human anatomy and experimental philosophy, and will gain an additional professor of chemistry, the consequent charges being borne in all these cases by Christ Church.

The Halley Lecture for 1914 will be delivered by Colonel C. F. Close, director of the Ordnance Survey, at the Examination Schools at 8.30 p.m. on May 20. Subject, "The Geodesy of the United Kingdom."

The celebration of the seven hundredth anniversary of the birth of Roger Bacon will be held on Wednesday, June 10.

BROWN UNIVERSITY, Rhode Island, is to receive a visit in November next from Prof. W. H. Bragg,